Optimization Model Research for Transportation and Distribution of Perishable Product Based on Indefinite Theory

Lele Wang

Xi’an International University

E-mail: yuapple2003@163.com

Abstract

In the paper, the production and distribution models under two circumstances are established according to production and distribution features of the perishable product with the uncertain product demand, punishment for early and late distribution time window and the deterioration rate of product. The research indicates that for perishable product, the supplier generally is more concerned about the deterioration rate of perishable product than the average loading rate and is willing to use more vehicles to prevent deterioration. More used vehicles will make the production time and the required transportation time for each vehicle shorter which will further cause less deterioration, but the vehicle start cost will increase, so it is required to weigh various parts in practice. The conditions considered in the Thesis can be further expanded and studied in case the different deterioration rates of different products are considered.

Keywords: Indefinite Theory, Transportation and Distribution, Transportation and Distribution of Perishable Product

1. INTRODUCTION

In real life, the values of many products drastically decrease with the passage of time, such as the perishable products of milk, vegetables, etc. the effect or value of which rapidly decreases in production and selling links as time goes on, and these kind of products are collectively called perishable products. The universality and distinctive features of these products make the formulation of reasonable production – distribution strategies for perishable products become a problem needing urgent solution. In recent years, the production and distribution strategies of perishable products have received widespread emphasis of researchers (Belofilo et al., 2015). The perishable commodities easily deteriorate during production and delivery process, so once produced, the values will constantly decline during transportation or after delivery; the revenue of supplier depends on the product state upon reception (Fedegrueen et al., 1986). Therefore, the production and distribution of perishable products seriously affect the revenue of supplier. The rapid and continuous decline feature of the products makes the supplier have to postpone the production time but accelerate the delivery speed as far as possible. In addition, what is more important is that the perishable product must be delivered within the allowable time window for delivery. Both late arrival and early arrival will cause cost loss to the supplier (Ahumadaaand, 2011). The just-in-time production and distribution strategy is adopted
by more and more suppliers to satisfy the order demand of retailers and in this way, the profit loss of supplier caused by deterioration of perishable product can be reduced.

2. PROBLEM DESCRIPTIONS

The emphasis of traditional production scheduling is to determine the production schedule of product, so the optimized production process under delivery plan is not taken into consideration (Bartoszewicz and, 2013). In addition, the emphasis of general distribution plan is to minimize the transportation cost of product without affecting the production progress. Therefore, the comprehensive production scheduling and distribution plan become an important problem for perishable product in urgent need of further studying. In consideration of the characteristics of perishable product, it is necessary to formulate a comprehensive production scheduling and transportation route to make the supplier provide fresh product as well as satisfy the customer requirements.

The mathematical model for production and distribution of some perishable product is proposed in the Thesis and in the model, the supplier must determine the production quantity, starting time and the best transportation route to deliver the product to the retailer (Jedermann et al., 2009). The target of supplier is to minimize the cost including production cost, shortage cost, deterioration cost of product, cost exceeding time window and operation cost of distribution route.

In case there is only one production line for the supplier but various kinds of perishable products will be produced, only one kind of product can be produced at the same time. Considering that there is only one production line, for batch production including several kinds of products, it is required to arrange the sent vehicle according to the production sequence of products and the products with high deterioration rate will take precedence (Li and Zhuang, 2009; Huang et al., 2011; Keilson and Seidmann, 1990). The processing time in machine for each kind of product and the initial price of product are given; the relevant switching time from one product to another required by the machine will not be considered. The required quantity of each kind of product is unknown, but short supply which will cause shortage cost is allowed. Various raw materials of producers can be promptly supplemented and the raw materials will not deteriorate. In case supply exceeds demand after arrival of products, the deterioration cost will be incurred.

3. MODEL CONSIDERING DETERIORATION RATE AND PUNISHMENT FOR BEING LATE

It is required to deliver the perishable product to retailers with different requirements for time window, so the flexible time window is adopted in the section; any late vehicle will be punished and any early vehicle must wait until the beginning of time window. In addition, there is load limit for each vehicle. To facilitate calculation, the parameters used in the Thesis are as follows (Chiang, 2010; Anonymous, 1946).

$$\min TC = \sum_{n \in N} \left[ \delta_{n1} \max \{Q_{n} - D_{n} + 0\} + \delta_{n2} \max \{D_{n} - Q_{n} + 0\} \right]$$

$$+ \sum_{m \in M} c_{m} Q_{m} + p \sum_{\sigma \in S} C_{\sigma} X_{\sigma} + g_{2} \sum_{i} \max \{ai - li, 0\}$$

(1)
\[
\sum_{j} \sum_{k} x_{ijk} = 1, i = 1, \ldots, R 
\]  
(2)

\[
\sum_{j} \sum_{k} x_{ijk} = 1, j = 1, \ldots, R 
\]  
(3)

\[
\sum_{i} x_{hjk} - \sum_{j} x_{ijk} = 1, h = 1, \ldots, R; k = 1, \ldots, R 
\]  
(4)

\[
\sum_{j} x_{0jk} \leq 1, k = 1, \ldots, R 
\]  
(5)

\[
x_{ijk} \in \{0, 1\}, i, j = 0, \ldots, R, i \neq j, k = 1, \ldots K 
\]  
(6)

Product scheduling constraint:
\[
\bar{t}_{k,n} \leq t_{k+1}, k = 1, \ldots, K - 1 
\]  
(7)

Vehicle load constraint:
\[
\sum_{ijn} Q_{ijn} x_{ijk} \leq Q_{veh}, \quad k = 1, \ldots, K 
\]  
(9)

Definition constraint:
\[
a_j = \max\{a_i, e_i\} + s_i + c_{ij} 
\]  
(10)

\[
a_j = \bar{t}_{k,n} + c_{0j}, \text{ when } x_{ijk} = 1, j = 1, \ldots, R; k = 1, \ldots, K 
\]  
(11)

\[
\bar{t}_{k,n} = t_{k,n} + \sum_{ijn} Q_{ijn} x_{ijk} 
\]  
(12)

\[
t_{k,n+1} = \bar{t}_{k,n} 
\]  
(13)

Nonnegative constraint:
\[
Q_{in} \geq 0, i = 1, \ldots, R; n = 1, \ldots N 
\]  
(14)

\[
t_{1,1} \geq 0 
\]  
(15)
Target function (1) indicates the minimum expected total cost of supplier. The first part in target function indicates the deterioration cost for surplus product of supplier when the demand of retailer is lower than the supply of supplier; the second part indicates the shortage cost of supplier when the demand of retailer is higher than the supply of supplier; the third part and fourth part respectively indicate the production cost and transportation cost; the last part indicates the punishment when the arrival time of transport vehicle exceeds the time window of retailer. Constraints (2) to (6) are the flow conservation limits for routes of traditional transport vehicles. Constraint (2) requires that Retailer i can only dispatch one vehicle every time. Constraint (3) describes that only one vehicle can arrive at the place of Retailer j every time. Constraint (4) indicates that for each Retailer h, the incoming vehicle must finally leave the node. Constraint (5) designates that each vehicle leaves the supplier as soon as possible every time. Constraint (6) designates $x_{ijk}$ as the integer variable of 0-1. Constraints (7) and (8) describes that the finishing time for production of all products in a vehicle must be earlier than the beginning time of production for products in the next vehicle or the closing time of time window for supplier (l0). Constraint (9) is the load limit of vehicle. Constraints (10)-(14) are the definition limit. Constraints (10) and (11) define the time for arrival Retailer j (when i is 0, it indicates the supplier; $c_{0j}$ indicates the time to reach Retailer j from the place of supplier). Constraint (10) ensures the elimination of subpath, because it is pointed out that the time to reach the next node must be greater than the time to reach the last node which is impossible for each node in subpath. Constraint (12) indicates the time to finish production of Commodity n in Vehicle k. Constraint (5.13) requires that for the same Vehicle k, the time to begin production of Commodity n+1 shall be consistent with the time to finish production of Commodity n. Constraints (14) and (15) are nonnegative constraints.

**4. ALGORITHM DESIGN**

The target function (1) is a NP-hard model. Therefore the variables $\{Q, t, x\}$ in $k, l, i, j, k$ are divided into two groups: $\{Q, t\}$ and $\{x\}$. The first combination is related to the time scheduling of product and the second group belongs to VRPTW, so target function (1) can be divided to two relevant problems for respective solution.

**4.1 Solution to production scheduling problem**

For the product scheduling problem of the first combination, the solution method based on partheno-genetic algorithm is adopted. With regard to the shortcomings of traditional genetic algorithm in solving the combinational optimization problem and constraint optimization problem, the monogenetic reproduction method is adopted by partheno-genetic algorithm and the crossover operator of traditional algorithm is cancelled (Zanoni and Zavanella, 2007; Coelho and Laporte, 2013). It is of higher efficiency in combinational optimization problem such as solving traveling salesman problem, production scheduling problem and model clustering problem. Compared with the traditional algorithm, the partheno-genetic algorithm is of simple operation and high calculation efficiency.

1) Encoding scheme and number of iterations

The encoding method by serial number is adopted by partheno-genetic algorithm; for the Thesis, suppose that there is only one production line and the serial number for n products are $a_1, a_2, \ldots, an$. Judge whether the number of iterations reaches the set algebra and in case this is the case, select the solution corresponding to the
chromosome with the best performance as the output of optimal solution for the production scheduling problem.

2) Fitness function

The production scheduling problem is to obtain the minimum value of target function and the selection operation of partheno-genetic algorithm keeps the individual with the greatest fitness, so the target function cannot be directly used as the evaluation function. Therefore, the evaluation function fn(ak) can be defined as: \[ f_n(a_k) = \frac{1}{1 + TC(a_k)} \]
where \( a_k \) indicates some kind of production scheduling scheme and \( TC(a_k) \) indicates the expenses corresponding to the scheme.

3) Genetic operator

The genetic operation of partheno-genetic algorithm includes gene transplacement, gene inversion, gene displacement, etc. The multi-point gene transplacement operator is adopted in the algorithm. Meanwhile, the new individual produced by mutation operation can be obtained at one time or several times through the previous gene transplacement operator, so the mutation operation is omitted. The selection method for championship is adopted in the algorithm, and the first \( N \) with good fitness will be selected to enter into the next generation after comparison of \( N \) individuals in species group and \( N \) individuals in parent species group after genetic operation.

4.2 Solve VRPTW

Step 1: input data. The production time of product is obtained from calculation of production scheduling problem and then input the product quantity and production time.

Step 2: add the retailer with the minimum insertion cost to the current existed route or establish one new route. Calculate the insertion cost of each retailer. Add the retailer with the minimum insertion cost to best position of current existed route. In case the capacity limit of vehicle cannot be satisfied or the insertion cost is higher than the cost to unload a new vehicle, establish a new route. Repeat Step 2 until all the retailers have arranged vehicles.

Step 3: route optimization. All the common route optimization methods in current vehicle route problem can be adopted; the insertion method is adopted in the Thesis, any insertion of new node will affect the arrival time for all products in transit.

4.3 Experimental analysis

The retailer information (including the place, demand, time window and service time) is based on Solomon problem. According to the relevant literatures, the following calculation examples are designed based on features of problem: five kinds of perishable food produced by some food processing plant, the corresponding different production time, production cost and deterioration cost. The tested retailers are selected from 100 retailers in problem set.
First, for deterioration rate analysis of product, the deterioration rate of all products ranges from 0.005 to 0.185. It can be concluded from Figure 1 that when the deterioration rate increases, the target value increases and especially when the deterioration rate of product increases to 0.1 and above, the increasing range of distribution cost becomes larger and the supplier would rather keep short supply than easily make the supply exceeds demand. Figure 2 indicates the convergence process for adoption of partheno-genetic algorithm curve with popsize=50 and gen=80. It is concluded that the algorithm is of relative better convergence and effectiveness. After 30 iteration, the convergence is fundamentally realized and the optimal result of comprehensive cost reaches 52.

**Figure 1.** The target relative to changes in the rate of corruption

**Figure 2.** Hand catenary process

**5. ADDITIONALLY CONSIDER VEHICLE START COST AND MODEL OF EARLY PUNISHMENT**

For the above problems, the vehicle start cost is not considered in target function, so only if the number of dispatched vehicles is more and the delivered time of product is short, the deterioration cost and the total cost are small. In fact, there is the corresponding start cost for each vehicle, so the target function will be greatly affected with the increase of quantity for the started vehicles. In addition, to prevent deterioration and punishment for being late, the arrival time may be earlier than the required time window of retailers with the increase of vehicle quantity; although cost of punishment for early arrival is lower than that for late arrival, early arrival will still affect the target cost. Based on that and considering the vehicle start cost and punishment for early arrival, the target function is as follows:
\[
\min Z = \sum_{i} U + \sum_{a} c_{a} Q_{a} + p \sum_{a} c_{a} x_{a} + \sum_{i} \left[ \delta_{s, i} \max \left\{ Q_{s} - D_{s}, 0 \right\} + \delta_{s, a} \max \left\{ D_{a} - Q_{a}, 0 \right\} \right] \\
+ g \sum_{i} \max \left\{ e_{i} - a_{i}, 0 \right\} + g \sum_{a} \max \left\{ a_{i} - l_{i}, 0 \right\}
\] 

(16)

Target Function (16) indicates the effort to make the expected total cost of supplier minimum. The first part of target function is the vehicle start cost; the second and third parts are respectively the production cost and transportation cost; the fourth part indicates the deterioration cost incurred by the surplus product when the demand of retailer is lower than the supply of the supplier; the fifth part indicates the shortage cost of supplier when the demand of retailer is higher than the supply of the supplier; the sixth part is the cost of punishment when the arrival time of transport vehicle at the place of retailer is earlier than earliest beginning time of time window; the seventh part is the cost of punishment when the arrival time of transport vehicle exceeds the limit of the retailer’s time window.

The target function in the Thesis is actually composed of production scheduling and vehicle route, so the ant colony optimization is adopted to first obtain the solution of vehicle route problem and then substitute the production scheduling for product obtained in simplex method of Nelder-Mead to obtain the solution of target function.

5.1 Improved ant colony algorithm

Substitute artificial ant for vehicle for distribution to the retailer; when the ant selects the following Retail Outlet j at Retail Outlet i, there are two main factors to be considered: the first is the level of closeness for relation between i and j which is called visibility and recorded as \( \eta_{ij} \); the second is the concentration of pheromone which is the feasibility from i to j reflected by the route scheme obtained from the completed circulation of ant and is recorded as \( \tau_{ij} \). At the time \( t \), the probability for Ant \( k \) transferring from Retail Outlet i to Retail Outlet j:

\[
p_{ij}^{k} = \begin{cases} \frac{\tau_{ij}^{a}(t)\eta_{ij}^{\beta}(t)}{\sum_{s \text{ allowed}} \tau_{ij}^{a}(t)\eta_{ij}^{\beta}(t)}, & j \text{ allowed} \\ 0, & \text{otherwise} \end{cases}
\]

(17)

The improvement made in the chapter is mainly the improvement to state transition probability formula. Ant colony optimization is the organic combination of positive feedback principle of information and the heuristic algorithm and the design of \( \eta_{ij} \) is critical to ant colony optimization. When the colony size is relatively large, it is hard to obtain the optimal solution in short time; in case the randomly produced amount of information for some route changes in excessively fast way, the searching stop can easily occur; to control the changing speed for amount of information, the following method will be adopted to select the following customer to be visited; introduce a new constant: \( q_{0} \in [0,1) \); before Ant \( k \) select route every time, \( q \in [0,1) \) is randomly generated; Ant \( k \) will according to the following equation to select the Route \( s \):

\[
j_{i(k)} = \begin{cases} \arg \max \left\{ \tau_{ij}^{a(k)}\eta_{ij}^{\beta(k)} \right\}, & q \leq q_{0} \\ p_{ij}, & q > q_{0} \end{cases}
\]

(18)
When $q_0 > q_t$, it is the exploratory search in the basic ant colony algorithm; when $q_0 \leq q_t$, the route with largest probability will be found as the selection from the obtained result and the ant selects the shortest route according to $q_0$ which is the deterministic search. Through appropriate adjustment of $q_0$, the deterministic search and exploratory search can be reasonably coordinated to accelerate the convergence speed of ant colony algorithm. In the early iteration of algorithm, the relatively great initial value is selected for $q_0$ to conduct deterministic search with the relatively large probability, so the speed to search the relatively optimal route in part can be accelerated; in the middle period of algorithm, the relatively small value is adopted for $q_0$ to increase the probability of exploratory search and then to expand the search space; in the late period of algorithm, the initial value of $q_0$ shall be recovered to accelerate the convergence speed.

5.2 Experimental results

![Figure 3. Hand catenary process](image3)

![Figure 4. The influence of vehicles number on average loading rate](image4)

The calculation example data is the same as those in Section 3 and the experimental results are shown in Figure 3 to Figure 5. Figure 3 indicates the convergence process with the improved ant colony optimization with popsize=60 and gen=80. With the ant colony optimization in the Thesis, the parameters are as follows: there are 30 ants in a colony; respectively take 1 and 3 for $\alpha$ and $\beta$; the optimal solution 75 will be found after calculation for 22 times. The value scope for quantity of vehicles is (2, 11) and
for a problem with given quantity of vehicles, the average loading rate of these vehicles under the corresponding time window can be calculated as shown in Figure 4. More vehicles and shorter scheduling time and transportation time of each vehicle may result in less deterioration which can rapidly reduce the total target cost; but the vehicle start cost is considered in target cost, so when the quantity of vehicles constantly increases, the total cost cannot be always reduced which can, in return, results in cost increase as shown in Figure 5. The cost increase is generally accompanied with the relatively low loading rate of vehicle which is in close relation with Figure 5.

![Figure 5](image.png)

**Figure 5.** The influence of vehicles number on target value

So when the vehicle quantity, deterioration rate and total cost are weighed, in general, the deterioration rate will be preferentially considered for fresh food; especially when the retailer makes requests for both the freshness of product and time, the supplier will preferentially consider the deterioration of product and add vehicles; but when the deterioration rate and time window are in the controllable scope, the supplier will additionally consider the loading rate of vehicle and vehicle start cost to avoid increasing trend of total cost.

### 6. CONCLUSIONS

In the Thesis, the production and distribution models under two circumstances are established according to production and distribution features of the perishable product with the uncertain product demand, punishment for early and late distribution time window and the deterioration rate of product. First, the vehicle start cost is not considered, but the cost of punishment for late arrival of vehicle and the deterioration cost of product are considered; under the circumstance, the partheno-genetic algorithm is adopted to solve the code of product scheduling and the traditional insertion point method is adopted to solve the vehicle route problem of retailer; the results indicate that the model and the method can effectively solve the combination problem of production scheduling and product distribution under the same framework; meanwhile, the algorithm adopted in the chapter applies to other fields of perishable product with the similar deterioration trend. Second, both the vehicle start cost and the cost of punishment for early and late time window of vehicle are considered; the improved ant colony optimization is adopted to solve the vehicle route problem with time window and the experiment indicates that the total cost decreases before increase with constant increase of vehicle quantity. The research indicates that for perishable product, the supplier generally is more concerned about the deterioration rate of perishable product than the average loading rate and is willing to use more
vehicles to prevent deterioration. More used vehicles will make the production time and the required transportation time for each vehicle shorter which will further cause less deterioration, but the vehicle start cost will increase, so it is required to weigh various parts in practice. The conditions considered in the Thesis can be further expanded and studied in case the different deterioration rates of different products are considered.

References


