Mechanical Analysis on the Lateral Action of Vehicle Loads on Circular Road Curves

Qingguo Yang, Tao Zhang, Jia Hu, Rong Yang, Congling Bao

School of Civil Engineering, Chongqing Jiaotong University, 66 Xuefu Road Nanan District, Chongqing 400074, China;

Abstract

Under the action of centrifugal force, when a vehicle runs on a road curve, the wheels of the vehicle usually produce lateral forces that act on the pavement. However, the prevailing design standards on asphalt pavement have not taken into consideration the influence of the lateral forces. Through lots of engineering practice, it is found that, under the action of vehicle load, the pavement is often more easily subject to pavement diseases in road curves than in straight sections. Focusing on circular road curves, this paper studies the lateral forces acting on the pavement, and puts forward the calculation formula of the lateral forces caused by centrifugal force. On the above basis, the design speed of vehicles is calculated, and the range of the lateral forces produced by the wheels acting on the pavement is given. The research results of the paper are of important guidance to the selection of load effect during the road design process.

Keywords: circular road curve, super elevation, lateral forces.

1. INTRODUCTION

When a vehicle runs on a road curve, it tends to slide towards the outer edge of the pavement under the action of centrifugal force (Chen et al., 2011; Liang, 2011; Liang and Xue, 2011). This tendency is bound to produce lateral forces on the pavement. This is particularly true to vehicles running at a high speed on a road curve with a small radius and a low super elevation, where the lateral forces must not be ignored (Liu et al., 2011; Tan et al., 2013; Xu et al., 2014). After an asphalt pavement has been put into operation for a few years, road curves are prone to pavement diseases, which affect traffic safety and comfort. For example, the asphalt pavement is pushed outward, often accompanied by rutting, longitudinal cracking and bulging (Liu et al., 2011; Liu, 2004).

Failing to quantify the influence of road curves in calculation analysis and design, the existing design and construction codes for asphalt pavement contain no specific provision or instruction on the lateral action of vehicle load, despite that the codes specify that the influence of road curves should be taken into consideration during material selection.

Based on the fact that centrifugal force needs to be considered if a vehicle runs on road
curves, many researchers have analyzed the action of vehicle load on pavement. For example, Liu (2004) deduces the formula for calculating the wheel load of inner and outer wheels in the direction perpendicular to the pavement (normal direction) under the action of centrifugal force, and points out that the existence of centrifugal force on road curves with super elevation runoff function is an important cause of pavement diseases, which more often than not occur at the inside of road curves. Xiao et al., (2008) elaborate the wheel load imbalance between inner and outer wheels, and describe the correlation between the normal forces of inner and outer wheels and the running speed, but they have not explored the lateral forces produced by the wheels.

The existing research has a certain reference value for further understanding of the stress state of the asphalt pavement of road curves. However, the following questions have not been answered or resolved: 1. How to illustrate the action of vehicle load on pavement? In the current code for bridge design, the action of vehicle on road curves of bridges is described as centrifugal force. Does the description suit the actual conditions of pavement and does it support direct design calculation (Wang, 2011; Wei and Liang, 2016; Xiao, 2011; Xing, 2009; Zhu, 2010)? 2. With no formula to calculate the lateral effect of vehicle load on pavement, the calculation lacks theoretical basis. 3. With no specified range of lateral action on pavement, it is impossible for engineering personnel to determine the value of the action or evaluate its effects.

To answer these questions, this paper studies the stress state of a vehicle running on a circular road curve with super elevation runoff function, deducts the formula for calculating the lateral forces produced by inner and outer wheels of such a vehicle, and provides the preliminary range of the lateral forces on pavement through the discussion of the actual lateral forces of wheels on pavement of highways of various grades when the vehicle is running at the design speed.

2. STRESS STATE OF PAVEMENT

In the General Code for Design of Highway Bridges and Culverts (China), the lateral action of vehicle load on curved bridges is called vehicle centrifugal force, which is obtained by multiplying the centrifugal force coefficient with the standard value of vehicle load. The calculation process indicates that the final vehicle centrifugal force acts on bridges in the form of a concentrated force (Shen et al., 2012; Sheng et al., 2014).

In light of the characteristics of pavement structure, the action on curved pavements can be called vehicle centrifugal force in qualitative description of the features of pavement structure. However, as the stress on pavements is significantly affected by the action area, it is improper to describe the action on pavements entirely as a concentrated force on highway bridges and culverts in the quantification or analysis of the response of pavement structure. Instead, it should be described as the lateral action on pavements by taking the ground contact area into account.

Because designers only need to verify the maximum lateral action, the lateral action
should be expressed either with centrifugal force (kN) and the ground contact area (m²), or the intensity of pressure (MPa) and action area of the centrifugal force. In short, the area is a factor not to be ignored.

3. ANALYSIS OF CENTRIFUGAL FORCE ON A VEHICLE RUNNING ON CIRCULAR ROAD CURVE

3.1 Mechanical Calculation Model

The author establishes a model of vehicle stress state (Figure 1) based on the standard axle load of single-axle double-wheel set BZZ-100. The model deducts the lateral action on pavement through analysis of the lateral action on the vehicle because the two actions are opposite reaction forces to each other.

**Figure 1. Mechanical Model of Single-axle Double-wheel Set**

In the figure, i is super elevation slope $\alpha$, B is inter-wheel spacing, H is height of vehicle gravity center, R is the curve radius, $f_i$ (i=1,2,3,4) are the lateral forces on the four wheels, $R_i$ (i=1,2,3,4) are the normal forces on the four wheels, F is the centrifugal force on the vehicle, G is the standard axle weight 100kN, and V is the running speed. In accordance with the single circular loadnomogram in the *Specifications for Design of Highway Asphalt Pavement (China JTG D50-2006)*, the author uses a circle (diameter=d) to illustrate the ground contact area of the two inner/outer wheels, and simplifies the calculation process into the model shown in Figure 2.

**Figure 2. Calculation Model for Wheel Sets on Road Curves with Super Elevation Runoff Function**
3.2 Derivation of Calculation Formula

Establish a coordinate system with the \( x \)-axis parallel to the curve slope, and the \( y \)-axis perpendicular to the slope. Based on the equilibrium relationship between \( x \) and \( y \) directions, and the moment equilibrium around the \( O \)-point, the following equations can be obtained:

\[
x - \text{axis}: f_i + f_o + G \sin \alpha = mv^2 \cos \alpha / R
\]

\[
y - \text{axis}: R_i + R_o = G \cos \alpha + mv^2 \sin \alpha / R
\]

\[
M_o(F): (f_i + f_o)H + R_iB / 2 = R_oB / 2
\]

The friction of the pavement against the wheels (i.e. lateral forces \( f_i \) and \( f_o \)) must not exceed the maximum static friction between the pavement and tires. Otherwise, the vehicle will slip laterally. Hence, the author establishes a supplementary equation:

\[
f_i + f_o \leq f_{\text{max}} = (G \cos \alpha + mv^2 \sin \alpha / R)\phi_x
\]

Where, \( \phi_x \) is the lateral adhesion coefficient, which is valued in relation to speed, pavement type and condition, wheel type, etc. The lateral adhesion coefficient of a dry pavement normally falls between 0.4 and 0.8.

From equations (1) and (4), the author deducts the critical speed of lateral slipping:

\[
V = \sqrt{\frac{127R(\sin \alpha + \phi_x \cos \alpha)}{\cos \alpha - \phi_x \sin \alpha}}
\]

In road design, \( \alpha \) is generally small. Setting \( \cos \alpha = 1 \) and \( \sin \alpha = \tan \alpha = i \), the author deducts the normal forces of inner and outer wheels on the basis of equations (1), (2) and (3):

\[
R_i = \frac{G}{B} \left[ \frac{B}{2} + Hi + \frac{V^2(iB/2 - H)}{127R} \right]
\]

\[
R_o = \frac{G}{B} \left[ \frac{B}{2} - Hi + \frac{V^2(iB/2 + H)}{127R} \right]
\]
normal force difference between inner and outer wheels affects the pavement and fail to give the formula to calculate the lateral forces (Liu, 2004; Xiao et al., 2008).

To obtain the lateral forces, the author continues with the deduction.

According to the rigid body motion theory in theoretical mechanics, the author set the speed of inner wheels as $v_i$, and the speed of the outer wheels as $v_o$, and provides the following relation:

$$v_i = \frac{R - B/2}{R} v$$
$$v_o = \frac{R + B/2}{R} v$$

(8)

By formulas (1) and (8), the author deducts the lateral forces on inner and outer wheels:

$$f_i = \frac{G}{2} \left[ \frac{R + B/2}{127 R^2} V^2 - i \right]$$

(9)

$$f_o = \frac{G}{2} \left[ \frac{R - B/2}{127 R^2} V^2 - i \right]$$

(10)

In the direction parallel to the curve slope, when the component of gravity and the component of centrifugal force balance each other out, i.e.:

$$G \sin \alpha = GV^2 \cos \alpha / 127R$$

(11)

The vehicle will pass through the road curve at a certain speed without producing any lateral action. Besides, the inner wheels and outer wheels will be subjected to the same amount of normal forces. If the vehicle speed is greater or lower than the certain speed, the normal forces on the inner wheels and outer wheels will not balance each other out, and the wheels will be subject to the action of lateral forces.

4. ANALYSIS OF THE INFLUENCING FACTORS ON VEHICLE’S LATERAL FORCES

To make a visual illustration of the variation law of lateral forces, the author probes into the relations between lateral forces and factors like speed, circular curve radius and super elevation slope.

In the calculation, let the inner and outer wheel spacing $B=1.8m$, and height of vehicle gravity center $H=1.6m$. According to the standard axis load, set $G$ at 100kN, the ground contact pressure of tire $P$ at 0.7Mpa, the equivalent diameter of pressure transfer surface
of tire at 0.302 m. Suppose: 1. the vehicle passes through the curve at a constant speed; 2. the ground contact area of the wheels remain the same during the whole process.

4.1 Relation between Lateral Forces and Speed

Taking the super elevation slope $i=4\%$ as an example, see Figure 3 for the relation between the lateral forces on inner and outer wheels and the speed when the radius of the curve the vehicle passes through $R=250 m$.

![Figure 3. Relation between Lateral Forces and Speed](image)

4.2 Relation between Lateral Forces and Curve Radius

See Figure 4 for the relation between the lateral forces on inner and outer wheels and the curve radius when the vehicle moves at the given speed $V=80 km/h$, and the super elevation slope $i=4\%$.

![Figure 4. Relation between Lateral Forces and Curve Radius](image)

4.3 Relation between Lateral Forces and Super Elevation

See Figure 5 for the relation between the lateral forces on inner and outer wheels and the
super elevation when vehicle moves at the given speed \( V = 80\text{km/h} \), and the curve radius \( R = 250\text{m} \).

![Graph showing the relation between lateral forces and super elevation.](image)

**Figure 5.** Relation between Lateral Forces and Super Elevation

From the above analysis, the author finds that the inner wheels produce a slightly greater lateral force than the outer wheels, but the difference is very small. The lateral forces increase as the speed increases and decrease as the curve radius/super elevation increases.

### 5. STUDY ON THE RANGE OF LATERAL FORCES

In road design, the curve radius and super elevation are usually determined in accordance with the highway grade and design speed. The higher the design speed, the larger the curve radius and super elevation. To explore the composite effect of speed, curve radius and super elevation on the lateral forces produced by wheels, the author conducts calculation and analysis by equation (9) and in accordance with the provisions on the minimum radius of circular curve specified in the *Technical Standard of Highway Engineering (China TJG B01-2014)*. In this way, the author obtains the lateral forces produced by wheels at different design speeds on highways of various grades. See Figure 6.

![Graph showing lateral forces under different design speeds.](image)

**Figure 6.** Lateral Forces under Different Design Speeds
Based on the above calculation results, when a vehicle runs normally at the design speed on a road curve, the super elevation has a great influence on the lateral forces (6~8kN) if the design speed is slow (i.e. small radius curve); however, if the design speed is fast (i.e. big radius curve), the super elevation has a small influence on the lateral forces (5kN).

6. CONCLUSION AND SUGGESTIONS

1) Under the action of centrifugal force, when a vehicle runs on a road curve with super elevation runoff function, it usually produces lateral forces that act on the asphalt pavement. The lateral forces are not to be ignored particularly when the vehicle runs rapidly on a road curve with a small radius and a high super elevation.

2) Due to the existence of super elevation, the inner wheels and outer wheels produce different amount of lateral forces. The lateral force produced by the inner wheels is often greater than that produced by the outer wheels.

3) According to calculation results, when the vehicle moves on the road curve at the design speed, the lateral forces produced by wheels fall between 6kN and 8kN. Hence, it is necessary to consider lateral forces produced by the wheels during the design of the asphalt pavement on road curves with super elevation runoff function. Of course, further research is needed on the value of lateral forces because speeding and overloading are not uncommon in actual driving.

7. REFERENCES

Wang Y.B. (2011). Design of Mountain Expressway Route Based on Operational Safety,