Risk Analysis of the Major Stock Markets Based on Cross-correlation and MF-DFA

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Abstract  
World widely efficient market theory is generally accepted by the present financial markets, but it cannot give the abnormal behaviours showed by markets a better explanation, especially in the forecasting of financial crisis. Based on daily return data of NASDAQ Composite Index (IXIC), the S&P 500 Index (^GSPC), the Hang Seng Composite Index Series, and the Shanghai Securities Composite Index (short for SSE), we carry out the cross-correlation research and risk analysis of four markets from 01 January 2008 to 08 February 2017, with multifractal detrended fluctuation analysis (MF-DFA). The results show that the cross-correlation is observed in these markets, which display long-term memory and multifractality. In addition, the SSE shows a more powerful long-term memory characteristic and greater volatility, which means a higher risk. It is also found that a possible risk transmission path is from U.S. to Hong Kong, then to Mainland China in the financial crisis period. Therefore, risk management, which are of critical significance for risk supervision and policy making, can be progressed further more.

Key words: Risk Analysis, Cross-correlation, Multifractal, Stock Markets.

1. INTRODUCTION  
With the development of international finance and economy, the international capital flows increasingly fast between various international financial markets; thus, the capital movements between different financial markets have been becoming increasingly close. Historically, if the financial markets in one country collapse, the financial markets in other related counties will correspondingly suffer from this collapsing (the Subprime Mortgage Crisis in U.S. and the Debt Crisis in Europe). Although the efficient market theory, which is first proposed by Fama (Fama, 1965), provides a widely applicable opinion, it cannot thoroughly explain many actual or statistical characteristics that appear in stock markets, such as the fat tail, long-term memory characteristics, volatility clustering, and self-similarity. For these visions, it is necessary to develop a new method to capture the characteristics of stock price fluctuations and the changing risk transmission in international financial markets, in order to perform better risk estimation, prevention and control.

2. LITERATURE REVIEW  
Mandelbrot first proposes fractal theory in the 1970s (Mandelbrot, 1977). In 1997, Mandelbrot proposes a multifractal model of asset returns to describe the variation of financial asset prices and he points out that the multifractal analysis can be reproduced volatile financial transactions and provide information on the predicted value of market trends, thus showing some regularity of various financial markets (Mandelbrot, Fisher and Calvet, 1997). It has been verified that multifractality widely exists in financial markets such as stock markets, future markets, spot markets, foreign exchange markets, derivative markets, interest rate markets and so on (Cajueiro and Tabak, 2005; Karuppiah and Los, 2005; Lee and Lee, 2007; Lim, Kim, Lee, Kim and Lee, 2007; Norouzzadeh and Jafari, 2005).

MF-DFA, short for multifractal detrended fluctuation analysis, is first proposed by Kantelhardt (Kantelhardt, Zschiegner, Koscielny-Bunde, Havlin, Bunde and Stanley, 2002), which can describe different statistical characteristics of time series on different time scales, also is an efficient way to test whether non-stationary time series is multifractal. Norouzzadeh et al. studies the Iranian Rial-US dollar exchange rate logarithmic variations through MF-DFA (Norouzzadeh and Rahmani, 2006). They find that the time series exhibits multifractality, which is mostly due to different long-range correlations for small and large fluctuations. Ying et al. measures multifractality in Shanghai stock market using MF-DFA (Ying, Zhuang and Jin, 2009) and finds that the generalized Hurst exponent can capture multifractality better. Panigrahi et al. characterizes price index behavior through fluctuation dynamics, involving companies listed on New York Stock Exchange.
By analyzing the daily returns of NASDAQ Composite Index by using MF-DFA method, it has been found that the return series does not fit the normal distribution and its leptokurtic, which indicates that a single-scale index is insufficient to describe the stock price fluctuation (Wang, Liu and Qin, 2014). An autocorrelation and cross-correlation of SENSEX fluctuations and Forex Exchange Rate is studied by Chen et al. (Chen, Miao, Tian, Ding and Li, 2016). They observe that the degree of short-term cross-correlation is higher than that in the long term and that the strength of multifractality after financial crisis.

However, most recent studies are focused on the original price series or its deformation in a single market; thus, they have not detected the connection and transmission of capital risks in international financial markets. This paper uses both the original return series and the reordered return series in order to study stock price fluctuations and discuss the risk transmission path in the major stock markets in U.S. and China, including NASDAQ, S&P 500, Hang Seng and SSE. Therefore, this paper may provide a better way to understand the risks in international financial markets.

3. METHODOLOGY

For series $\{x(i)\}$, where $i=(1,2,...,N)$ and $N$ is the length of $\{x(i)\}$, the MF-DFA method is as following.

Through the sum process, the original series $\{x(i)\}$ merges into a new series $\{y(j)\}$, with $\bar{x}$ indicating the mean value of series $\{x(i)\}$.

$$y(j) = \sum_{i=1}^{j}[x(i) - \bar{x}], i = (1,2,...,N)$$

(1)

Next, divide the new series $\{y(j)\}$ into $N_s = \text{int}\left(\frac{N}{s}\right)$ non-overlapping segments of equal length $s$. Usually time scale $s$ is not an integer multiple of length $N$, so repeating the same procedure from the opposite end to get the whole part of series $\{y(j)\}$ other than disregard extra parts. Therefore, total $2N_s$ segments are obtained.

$$y_{1,2}(j) = y(l + j), j = (1,2,...,s), v = (1,2,...,2N_s), l = (v-1)s$$

(2)

Fit local trend function $\tilde{y}_v(j)$ on $2N_s$ sub segments $v$ by the least squares method in order to eliminate the local trends in each sub segments $v$ and get the residuals series $e_v(j)$.

$$e_v(j) = y_v(j) - \tilde{y}_v(j), j = (1,2,...,s)$$

(3)

Then calculate the mean squared value of $2N_s$ sub segments without local trends.

$$F^2(s,v) = \frac{1}{s}\sum_{j=1}^{s} e_v^2(j) = \frac{1}{s}\sum_{j=1}^{s} [y[(v-1)s + j] - \tilde{y}_v(j)]^2, v = (1,2,...,N_s)$$

(4)

$$F^2(s,v) = \frac{1}{s}\sum_{j=1}^{s} [y[N - (v - N_s)s + j] - \tilde{y}_v(j)]^2, v = (N_s + 1, N_s + 2,...,2N_s)$$

(5)

Also average all segments to get the $q-th$ order fluctuation function.

$$F_q(s) = \begin{cases} \frac{1}{2N_s}\sum_{v=1}^{2N_s} [F^2(v,s)]^{\frac{q}{2}}, q \neq 0 \\ \frac{1}{4N_s}\sum_{v=1}^{2N_s} \ln[F^2(v,s)], q = 0 \end{cases}$$

(6)

To any fixed $q$, determine the scaling exponent of fluctuation function, and the relationship between $F_q(s)$ and $s$ is obtained.

$$F_q(s) \propto s^{h(q)}$$

(7)

For every time scale $s$, we can get a correspondent value $F_q(s)$. The $q-th$ order generalized Hurst
exponent is the slope of \( \ln(F_s(s)) \cdot \ln(s) \). Here, if \( h(q) \) is a constant and independent from \( q \), the series \( \{x(i)\} \) is nonfractal; and if \( h(q) \) is a function of \( q \), the series \( \{x(i)\} \) is multifractal.

The multifractal spectrum \( f(\alpha) \) is another efficient way to describe the multifractal time series. The \( h(q) \) generated by MF-DFA is related to Renyi exponent \( \tau(q) \).

\[
\tau(q) = qh(q) - 1
\]  
\[ \tag{8} \]

Thus, the multifractal spectrum \( f(\alpha) \) can be generated by formula (9) and (10).

\[
\alpha = h(q) + q h'(q)
\]  
\[ \tag{9} \]

\[
f(\alpha) = q(\alpha - h(q)) + 1
\]  
\[ \tag{10} \]

4. DATA DESCRIPTION

The adjusted daily closing prices of four major stock markets from U.S., Hong Kong and China, from 1 January 2008 to 08 February 2017 are selected as the sample data. The major markets include including the NASDAQ Composite Index (^IXIC) and the S&P 500 Index (^GSPC) of U.S., the Hang Seng Composite Index Series of Hong Kong, the Shanghai Securities Composite Index (short for SSE) of Mainland China. All the data are derived from Yahoo! Finance and calculated in MATLAB R2013b.

Based on the original prices, we assume \( I_t \) represents the closing price in time \( t \); thus, the logarithmic rate of return \( R_t \) in time \( t \) is as formula (11).

\[
R_t = \ln(I_t) - \ln(I_{t-1})
\]  
\[ \tag{11} \]

Therefore, the daily logarithmic rates of return \( R_t(t = 1, 2, ..., N) \) in four stock markets are obtained. Series \( R_t \) can be converted to a random walk like series by formula (1). The multifractal properties of these random walks are reflected by their similarities, as illustrated in the lower panel of figure 1. Small “hills” and “valleys” with similar structure appear when you zoom on the large “hills” and “valleys” of the random walk (Peng, Havelin, Stanley and Goldberger, 1995).

![Figure 1](image)

Figure 1 Daily logarithmic rates of return \( R \) of NASDAQ Composite Index (IXIC), the S&P 500 Index (^GSPC), the Hang Seng Composite Index Series, and the Shanghai Securities Composite Index from 01 January 2008 to 08 February 2017.
Table 1 The fundamental statistics of $R$

<table>
<thead>
<tr>
<th>Stock Indices</th>
<th>Series</th>
<th>Mean</th>
<th>Median</th>
<th>RMS</th>
<th>Variance</th>
<th>Kurtosis</th>
<th>Skewness</th>
<th>J-B test</th>
</tr>
</thead>
<tbody>
<tr>
<td>NASDAQ</td>
<td>$R$</td>
<td>0.000340</td>
<td>0.000920</td>
<td>0.0142</td>
<td>0.000203</td>
<td>10.1157</td>
<td>-0.2622</td>
<td>1</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>$R$</td>
<td>0.000201</td>
<td>0.000572</td>
<td>0.0134</td>
<td>0.000180</td>
<td>13.0935</td>
<td>-0.3207</td>
<td>1</td>
</tr>
<tr>
<td>Hang Seng</td>
<td>$R$</td>
<td>0.000070</td>
<td>0.000070</td>
<td>0.0163</td>
<td>0.000266</td>
<td>12.3003</td>
<td>0.0593</td>
<td>1</td>
</tr>
<tr>
<td>SSE</td>
<td>$R$</td>
<td>0.000229</td>
<td>0.000285</td>
<td>0.0173</td>
<td>0.000300</td>
<td>7.3068</td>
<td>-0.5038</td>
<td>1</td>
</tr>
</tbody>
</table>

The table 1 illustrates the skewnesses of the four-return series are all not equal to 0 and the kurtoses are much larger than those of normal distributions, which should be approximately equal to 3. The fundamental statistics show that the return series is not normally distributed and has leptokurtic characteristics, indicating that traditional EMH is not a proper way to describe the return series.

However, the long-term memory characteristics of low or high volatility in time series also cause multifractal behaviors. In this paper, random reordering process is used to eliminate the data correlation and keep the volatility, demonstrating the volatility of reordered series is the same as the original one, without long-term memory characteristics. The reordered return series can be generated by the following random reordering procedures.

Firstly, to generate a random pair of natural numbers $(a, b)$, in which $a$ and $b$ are less than or equal to the length $N$ of time series.

Secondly, to change the $a$-th and the $b$-th number in time series.

Thirdly, to repeat the above two procedures $20 \times N$ times, in order to make sure the order fully disrupted.

5. EMPIRICAL RESEARCH

5.1. Cross-correlation Analysis

$x$ and $y$ are two functions. Cross-correlation is used to find how much $x$ must be shifted along the x-axis to make it identical to $y$. Also, cross-correlation can determine the time delay between two functions, where $m$ is the time lag.

$$R_{xy}(m) = E[x_{n+m}y'_n] = E[x_ny'_{n-m}]$$ (12)

$$R_{xy}(m) = \begin{cases} \sum_{n=0}^{N-m-1} x_{n+m}y'_n, m \geq 0 \\ 0, m < 0 \end{cases}$$ (13)

$$\hat{R}_{xy, coefficient}(m) = \frac{1}{N-m} \sum_{n=0}^{N-m-1} \frac{x_{n+m}}{\sigma_x} \frac{y'_n}{\sigma_y}$$ (14)

Here we select top 10 absolute values from cross-correlation functions in order to find the time delay between the pair market.
The time lag $m$ between U.S and HK is much less than it between U.S. and Mainland China; and $m$ of Hang Seng vs SSE is much smaller than it of U.S. vs Hang Seng. These results show that the most possible way for risk transmission is firstly from U.S. to Hong Kong, then to Mainland China.

5.2. Generalized Hurst Exponents under MF-DFA

The MF-DFA method is applied to the daily logarithmic rates of return $R$ and its reordered series $R_{\text{reordered}}$ in selected four stock markets. Here we define parameter $q$ is [-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10] and $s$ is an integer array, ranging from 3 to $\frac{N}{5}$, where $N$ is the length of $R$ and $R_{\text{reordered}}$. Table 2 is the result of MF-DFA, showing the generalized Hurst exponents $H_q$ of $R$ and $R_{\text{reordered}}$ in each stock market, when $q$ changes from -10 to 10.

<table>
<thead>
<tr>
<th>Order</th>
<th>$h(q)$ of NASDAQ</th>
<th>$h(q)$ of S&amp;P 500</th>
<th>$h(q)$ of Hang Seng</th>
<th>$h(q)$ of SSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>$R$</td>
<td>$R_{\text{reordered}}$</td>
<td>$R$</td>
<td>$R_{\text{reordered}}$</td>
</tr>
<tr>
<td>-10</td>
<td>0.6726</td>
<td>0.8256</td>
<td>0.6334</td>
<td>0.8046</td>
</tr>
<tr>
<td>-8</td>
<td>0.6573</td>
<td>0.8043</td>
<td>0.6187</td>
<td>0.7842</td>
</tr>
<tr>
<td>-6</td>
<td>0.6361</td>
<td>0.7725</td>
<td>0.5989</td>
<td>0.7539</td>
</tr>
<tr>
<td>-4</td>
<td>0.6063</td>
<td>0.7222</td>
<td>0.5719</td>
<td>0.7079</td>
</tr>
<tr>
<td>-2</td>
<td>0.5632</td>
<td>0.6438</td>
<td>0.5362</td>
<td>0.6391</td>
</tr>
<tr>
<td>0</td>
<td>0.5159</td>
<td>0.5639</td>
<td>0.5058</td>
<td>0.5508</td>
</tr>
<tr>
<td>2</td>
<td>0.5151</td>
<td>0.5177</td>
<td>0.4949</td>
<td>0.4977</td>
</tr>
<tr>
<td>4</td>
<td>0.4946</td>
<td>0.4799</td>
<td>0.4511</td>
<td>0.4534</td>
</tr>
<tr>
<td>6</td>
<td>0.4644</td>
<td>0.4480</td>
<td>0.4112</td>
<td>0.4213</td>
</tr>
<tr>
<td>8</td>
<td>0.4406</td>
<td>0.4224</td>
<td>0.3847</td>
<td>0.3990</td>
</tr>
<tr>
<td>10</td>
<td>0.4234</td>
<td>0.4030</td>
<td>0.3668</td>
<td>0.3829</td>
</tr>
<tr>
<td>$\Delta h$</td>
<td>0.2492</td>
<td>0.4226</td>
<td>0.2666</td>
<td>0.4217</td>
</tr>
</tbody>
</table>

**Figure 3** The generalized Hurst exponent $h(q)$ of $R$ and $R_{\text{reordered}}$ in each stock market is not a constant but the function with respect to $q$, indicating multifractality in $R$ and $R_{\text{reordered}}$.

As can be seen from table 2, when $q$ changes from -10 to 10, the $h(q)$ of original NASDAQ return series descends from 0.6726 to 0.4234; the $h(q)$ of original S&P 500 return series descends from 0.6334 to 0.4030; the $h(q)$ of original Hang Seng return series descends from 0.9764 to 0.3921; and the $h(q)$ of original SSE return series descends from 1.8873 to 0.5070. The $h(q)$ of four stock market indices obviously are not a constant, indicating that there are distinct multifractal structures in these four markets. So, using nonfractal model to describe is not appropriate.

When $q$ is a negative or relatively small positive number, $h(q) > 0.5$. The small fluctuations of the rates of return are amplified, expressing the persistent feature. Correspondingly, when $q$ is a relatively large positive number, $h(q) < 0.5$, indicating that the large fluctuations are dominant; therefore, anti-persistent feature is clear.
For given positive order $q$, the $h(q)$ of original SSE return series is higher than it of original Hang Seng one, and the $h(q)$ of original Hang Seng return series is higher than it of original NASDAQ or S&P 500 one. However, for any given negative order $q$, there is no distinct difference in the $h(q)$ of the original return series in these four stock markets. This phenomenon interprets that there is the strongest state persistence and the weakest state anti-persistence in SSE and the second strongest state persistence and the second weakest state anti-persistence in Hang Seng stock market. The state persistence in NASDAQ and S&P 500 stock markets is much weaker than it in Chinese ones; oppositely, the state anti-persistence in U.S. stock markets is the strongest. According to the variation range of $h(q)$, from more distinguished to less distinguished multifractal feature the stock markets are, in order, SSE, Hang Seng, S&P 500 and NASDAQ.

After reordered, the $h(q)$ of SSE decreases the most sharply, especially for any positive $q$; then following the Hang Seng, S&P 500 and NASDAQ stock markets. Moreover, comparing the $h(q)$ of the reordered return series in four stock markets, the $h(q)$ in SSE is the closest to 0.5, indicating that the correlation between $h(q)$ and the reordered return series in SSE is the strongest.

For these four stock markets, the $h(q)$ changes insignificantly with $q$ in both the original and reordered return series. The multifractal feature is much weakened. The reordering procedure eliminates the multifractality caused by long-term memory characteristics to a great extent.

5.3. Multifractal Spectrum Analysis

Furthermore, we analyze the multifractal feature in these four stock markets with multifractal spectrum based on MF-DFA. Through generalized multifractal model, Koscielny-Bunde finds the relationship between generalized Hurst exponent $h(q)$ and order $q$ can be fitted, as formula (16) (Koscielny-Bunde, Kantelhardt, Braun, Bunde and Havlin, 2006). This illustrates that $h(q)$ can be described by two independent parameters when it is infinite.

$$h(q) = \frac{1}{q} \ln \left[ \frac{\ln (a^q + b^q)}{\ln 2} \right]$$ \hspace{1cm} (15)

With formula (8), we can get the fitted formula (16) between $\tau(q)$ and $q$.

$$\tau(q) = -\frac{\ln (a^q + b^q)}{\ln 2}$$ \hspace{1cm} (16)

The parameter $a$ and $b$ can describe the multifractal strength in a time series. According to formula (16), we fit the $\tau(q)$ of both original and reordered return series in each stock market, shown in table 3. Here is a sample $\tau(q)$ fitting figure of NASDAQ market.

**Figure 4** The fitting figure of $\tau(q)$ based on the original return series of NASDAQ, where $a = 0.7947$, $b = 0.5881$, and $R^2 = 0.999978$. The fitting result is very good.

In multifractal model, the strength of a time series can be described by $\Delta \alpha$. The bigger the $\Delta \alpha$ is, the stronger the multifractality is.

$$\Delta \alpha = \alpha_{\text{max}} - \alpha_{\text{min}} = h(-\infty) - h(+\infty) = \frac{\ln a - \ln b}{\ln 2}$$ \hspace{1cm} (17)
Similarly, we calculate the $\Delta\alpha$ of each return series, shown as table 3.

**Table 3** The multifractal strength of $R$ and $R_{\text{reordered}}$ in NASDAQ, S&P 500, Hang Seng and SSE

<table>
<thead>
<tr>
<th>Stock Indices</th>
<th>Series</th>
<th>$a$</th>
<th>$b$</th>
<th>Sum of Squares for Error</th>
<th>$R^2$</th>
<th>$\Delta\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NASDAQ</td>
<td>$R$</td>
<td>0.7947</td>
<td>0.5881</td>
<td>0.002954746</td>
<td>0.999978055</td>
<td>0.4343</td>
</tr>
<tr>
<td></td>
<td>$R_{\text{reordered}}$</td>
<td>0.8113</td>
<td>0.5278</td>
<td>0.013454195</td>
<td>0.99992109</td>
<td>0.6202</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>$R$</td>
<td>0.8261</td>
<td>0.6031</td>
<td>0.008964859</td>
<td>0.999921105</td>
<td>0.4539</td>
</tr>
<tr>
<td></td>
<td>$R_{\text{reordered}}$</td>
<td>0.5351</td>
<td>0.824</td>
<td>0.014126768</td>
<td>0.99991448</td>
<td>0.6228</td>
</tr>
<tr>
<td>Hang Seng</td>
<td>$R$</td>
<td>0.4079</td>
<td>0.8175</td>
<td>0.073120977</td>
<td>0.999666723</td>
<td>1.0030</td>
</tr>
<tr>
<td></td>
<td>$R_{\text{reordered}}$</td>
<td>0.8487</td>
<td>0.479</td>
<td>0.073461281</td>
<td>0.999627527</td>
<td>0.8252</td>
</tr>
<tr>
<td>SSE</td>
<td>$R$</td>
<td>0.763</td>
<td>0.2502</td>
<td>0.439656452</td>
<td>0.999357175</td>
<td>0.4408</td>
</tr>
<tr>
<td></td>
<td>$R_{\text{reordered}}$</td>
<td>0.7684</td>
<td>0.5661</td>
<td>0.052096274</td>
<td>0.999676244</td>
<td>0.4408</td>
</tr>
</tbody>
</table>

As can be seen from table 3, how $h(q)$ changes with $q$ can be well described by parameters $a$ and $b$, and all the fittings are in 95% confidence intervals. Also, the $\Delta\alpha$ of each series directly demonstrates that the multifractality of the original return series is stronger than it of the reordered one, especially in the Chinese stock markets. The result of $\Delta\alpha$ is consistent with figure 3, where the variance range of $h(q)$ shows the multifractality.

![Figure 5](image)

**Figure 5** The $\tau(q)$ of $R$ and $R_{\text{reordered}}$ in each stock market

Next, $\tau(q)$ of each return series can be obtained, shown as figure 5. For nonfractal, $\tau(q)$ is linear; oppositely, for multifractal, $\tau(q)$ changes with $q$ non-linearly. The stronger the nonlinearity of $\tau(q)$ is, the stronger the multifractality in the time series is. For the whole four stock market indices, the nonlinearity of $\tau(q)$ in the reordered return series is much weaker than it in the original series. It also illustrates the interpretation of figure 4 and table 2.

To compare the correlationship among these four stock markets, we calculate the correlation coefficients $r$ of the pair markets, based on the $h(q)$ and $\tau(q)$ of the original return series, shown as table 4.

**Table 4** The correlationship between pair stock markets based on $R$

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau(q)$</td>
<td>0.9997</td>
<td>0.9952</td>
<td>0.9862</td>
<td>0.9971</td>
<td>0.9895</td>
<td>0.9976</td>
</tr>
<tr>
<td>$h(q)$</td>
<td>0.9894</td>
<td>0.9692</td>
<td>0.9273</td>
<td>0.9555</td>
<td>0.8970</td>
<td>0.9869</td>
</tr>
</tbody>
</table>

Although all the correlation coefficients are nearly over 95%, the small differences can still be obtained from table 4. Comparing the coefficients in American and Chinese stock markets, there is a little stronger correlation between American and Hong Kong than it between American and Mainland China. For U.S., domestically NASDAQ and S&P 500 are very strong correlated. The correlationship between Hong Kong and Mainland China is stronger than between American and Mainland China. Even though a part of risks can transmit from U.S. to Mainland China directly, some parts of risks will transmit in a different way. The risk transmission path can be simply described as U.S. – Hong Kong – Mainland China.
The fundamental reason of this path is the imperfect and relatively closed financial system in China, which causes less correlation with American markets. However, Hang Seng stock market is highly connected with NASDAQ and S&P 500, because they are relatively open. Meanwhile, Hang Seng stock market is affected by the other major stock markets in the worldwide, such as Europe and Japan. It is the result of combined effect of the global financial markets.

According to formula (9) and (10), the multifractal spectrums of $R$ and $R_{\text{reorder}}$ in these four stock markets can be captured, shown as figure 6. Each multifractal spectrum width of the reordered return series is less than it of the original one, illustrating the interpretation of table 3. The variety of reordered return series confirms that persistent relevance is an important factor to the multi-scaling changes in price volatility.

The width of SSE is the widest also means that its original return series has the strongest multifractality. The previous fluctuations will have longer persistent impact on the inefficient market. When the NASDAQ or S&P 500 stock market suffers severe shocks, its recovery speed will be much higher than Hang Seng and SSE. Especially, SSE will have to go through an unpredictable fluctuation in a long period. As can be seen from the multifractal spectrums of $R_{\text{reorder}}$, the fluctuation in each stock market without long memory impaction is similar, indicating that the cycle of stock markets is healthy. However, the state persistence in each market will cause multifractality, and indirectly amplify the risks.

![Figure 6](image)

**Figure 6** Multifractal Spectrums $f(\alpha) \square \alpha$ of the four stock markets

6. CONCLUSIONS

The multifractal model, which is acknowledged to scale the financial markets, can capture more detail information in stock markets. MF-DFA, which is executed in this paper, not only captures the multifractality, but also contributes to the detection of risk transmission path in international stock markets.

Through the statistical research on NASDAQ, S&P 500, Hang Seng and SSE using MF-DFA, it is discovered that in the financial crisis period a risk transmission path is from U.S. to Hong Kong, then to Mainland China. The risks of American financial markets first spread in domestic markets; then through the international financial markets, the risks spread to the markets in Hong Kong. As a connection point, Hong Kong transferred the risks of international financial markets to Mainland China. The empirical results also suggest that the long-term memory characteristics of the return volatility are a main reason of multifractality in stock markets. The entire stock market is a process affected by large and small fluctuations in some periods, rather than a random process. The state persistence contributes to the multi-scaling changes in market volatility. The stronger the multifractality is, the more inefficiency in the financial market, interpreting that the inefficient market is hard to reach normal fluctuation after a severe shock.

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