The analysis of real estate investment value based on prospect theory--with an example in Hainan province

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Abstract

A decision-making method which considers double reference points and prospect theory is proposed. First, construct the positive and negative value matrix based on the double reference points (positive ideal point and zero point), then, the weight of every criterion is obtained by using the entropy method, thus, the decision-making weight when faced with gains and losses is acquired, and all the projects are sorted according to the comprehensive prospect value based on prospect theory. Finally, the investment value of the real estate market in Hainan Province is evaluated to illustrate the feasibility and effectiveness of this method.

Keywords: Prospect theory, Entropy method, Multi-criterion decision making.

1. INTRODUCTION

The real estate industry is an iconic industry to reflect the social and economic development and people’s living standard of a country. The development of the real estate industry vigorously facilitates the sustainable development of a whole country’s economy. With the improvement of urban residents' living standard, the demands of house keep on increasing, and thus, the real estate industry develops rapidly. Generally, an investor will firstly makes a judgment of a city’s real estate market if he wants to invest in the city. Traditionally, an analysis of the level of housing price is a measurement of the feasibility to invest, however, this method has its limitation, the risk of investment is big. Therefore, it is of great significance to build a scientific and exercisable decision system of investment in real estate.

At present, most of the research on multi-criteria decision making under risk is based on expected utility theory (Tzeng and Huang, 1981), however, based on a series of experiments, Kahneman and Tversky indicate that people’s decision making behavior is not in line with the expected utility theory (Kahneman and Tversky, 1979). Therefore, prospect theory is proposed to describe decision making behavior. The theory says, in practical decision making, decision makers will pursue satisfaction maximization instead of utility maximization. Prospect theory is a descriptive decision-making model about risk preference (Tversky and Kahneman, 1992), which well considers the psychological factors of decision makers. Meanwhile, prospect theory is wildly used in risk decision (Jou and Chen, 2013; Zhang and Fan, 2013), finance (Li, 2005), capital asset pricing model (Barberis et al., 2001).

2. PROSPECT THEORY

2.1 Preliminaries

Prospect theory is mainly used to analyze problem from the perspective of gains and losses, namely, the attitude of decision maker is asymmetric when faced with gains and losses: the decision maker tend to risk aversion when faced with gains while tend to risk preference when faced with losses (Kahneman and Tversky, 1979). Prospect theory consists of two phases: The Editing Phase and the Evaluation Phase. During the first phase, the reference point which is a reference of the decision making outcome is determined. The Evaluation Phase is the key to the prospect theory, the theory adopts a classical “total value” to measure the outcome of the decision making. The description and determination of the total value is reflected by the combination of value function and decision making weight function created by Kahneman and Tversky.
The prospect theory breaks the classical “rational man” hypothesis, it explains people’s decision making behavior under uncertain condition from the perspective of people’s bounded rationality, and depicts people’s important psychological characters. And thus, some new enlightenments are provided for the decision-making research:

(1) In the decision making process, a reference point rather than the final wealth level is necessary to determine the gains or losses, and different reference point may change the decision making behavior.

(2) People’s preference may suffer from the inducement which is irrelevant with the selected object when making a selection.

(3) People are more sensitive to losses than gains, and this difference will influence the final decision making.

(4) The actual outcome of the early decision making will have an impact on the later risk attitude and decision making. The early profit will strength people’s risk preference, while the early losses will intensify the pains of the later losses, and the degree of risk aversion will correspondingly increase.

2.2 Prospect theory model

The prospect value is codetermined by the value function and weight function, namely:

\[ V = \sum_{i=1}^{n} w(p_i)v(x_i) \]  \quad (1)

Where \( V \) is the prospect value, and \( w(p) \) is the weight function which is the monotone increasing function of the probability assessment, and \( v(x) \) is the value function coming from the subjective feeling of the decision maker.

The value function proposed is a power function (Tversky and Kahneman, 1992), namely:

\[ v(x) = \begin{cases} 
  x^\alpha, & x \geq 0; \\
  -\theta(-x)^\beta, & x < 0
\end{cases} \quad (2) \]

Where \( x \) is the difference between actual utility and expected utility, \( x>0 \) represents gains while \( x<0 \) represents losses. \( \alpha \) and \( \beta \) show the concave-convex degree of the value power function of the gain and loss range, \( \alpha, \beta<1 \) shows the progressive decrease of sensibility, \( \theta \) shows that the loss region is more steeper than the gain region, \( \theta>1 \) represents the loss aversion.

In the research (Tversky and Kahneman, 1992), the outcome is more in line with the practical situation when \( \alpha=\beta=0.88, \theta=2.25 \).

The value function has three important characters:

(1) The gains and losses are relative to the reference point, and the reference point may be different in the same decision making;

(2) Investors are risk aversion when faced with gains while are risk preference when faced with losses;

(3) Compared with gains, investors are more sensitive to losses.

Kahneman and Tversky regard weight function as a subjective judgement of the decision maker based on the occurrence probability of event outcome, it is neither the probability nor the linear function of the probability. In some uncertain risk decision-making, the judgement probability may violate the binary complementary relationship. Therefore, it is necessary to consider the influence on account of the decision maker’s preference to uncertain event and the judgement of unknown probability event. The weight function is shown as follows:
\[ w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}} \]  
\[ w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{\frac{1}{\delta}}} \]  

Where \( w^+(p) \) represents the probability function of losses, \( w^+(p) \) represents the probability function of gains, \( \gamma \) represents the coefficient of gains, and \( \delta \) represents the coefficient of losses.

The decision weight \( w(p) \) is jointly generated by \( w^+(p) \) and \( w^-(p) \), it is a strictly increasing function in the interval \([0,1]\), and \( w(0)=0, w(1)=1 \).

The characters of the weight function are shown as follows:

1. The decision weight function is not the probability, and \( w \) is an increasing function of \( p \), however, not only does it not satisfy the probability axioms, but also can not be interpreted as the degree of personal expectation.

2. When the occurrence probability \( p \) is small, \( w(p) > p \), which illustrates that the decision maker puts excessive emphasis on the small probability event, however, when the occurrence probability is general or big, \( w(p) < p \), which illustrates that the decision maker overvalues the low probability event and ignores the events of the routine.

When faced with risk decision making, the decision maker will not consider their eventual wealth level, instead, a reference point is chosen to judge the gains or losses. Therefore, the decision making may change according to different reference points.

### 3. DECISION-MAKING METHOD

#### 3.1 The description of problem

Considering the multi-criterion project selection, suppose that \( A = \{a_1, a_2, ..., a_m\} \) is a set of the projects, where \( a_i \) is the \( i^{th} \) project, \( i=1, 2, ..., m \), and \( Q = \{q_1, q_2, ..., q_n\} \) is the criterion set, where \( q_j \) is the \( j^{th} \) project, \( j=1, 2, ..., n \), \( w=(w_1, w_2, ..., w_n)^T \) is the weight vector of the criteria, where \( w_j \) represents the weight of criterion \( q_j \), and \( w_j \geq 0 \), \( j=1, 2, ..., n \), \( \sum_{j=1}^{n} w_j = 1 \). The weight vector is given by entropy method in this paper.

#### 3.2 Decision-making method

With respect to multi-criterion decision making problem, the most common criterion types are efficient type and cost type. \( C=(c_{ij})_{m \times n} \) represents the decision making matrix, where \( c_{ij} \) represents the criterion value of project \( A_i \) to criterion \( q_j \). The main purpose of this paper is to select the best project from the project set \( A \) according to the decision making matrix \( C=(c_{ij})_{m \times n} \) and the criterion’s weight vector \( w=(w_1, w_2, ..., w_n)^T \). The specific steps are as follows:

1. The standardization of decision making matrix

Due to the different units of the criteria, it is necessary to normalize the decision making matrix \( C=(c_{ij})_{m \times n} \), suppose that the normalized decision making matrix is \( R=(r_{ij})_{m \times n} \), where \( i=1, 2, ..., m, j=1, 2, ..., n \), \( r_{ij} \) can be calculated by the following formula:

\[
 r_{ij} = \begin{cases} 
 (c_{ij} - \min(c_{ij})) / (\max(c_{ij}) - \min(c_{ij})), & q_j \in Q^1; \\
 (\max(c_{ij}) - c_{ij}) / (\max(c_{ij}) - \min(c_{ij})), & q_j \in Q^2; 
\end{cases} \quad i=1, 2, \cdots m, \quad j=1, 2, \cdots n
\]  

(5)
(2) The selection of reference point

The selection of reference point is the key of prospect theory, since the decision makers often evaluate the gains or losses of a project compared with the reference point. As for the selection of reference point, the decision maker put emphasis on the difference between outcome and expected outcome instead of the outcome itself. Therefore, it is of great importance to select the reference point.

In this paper, the positive ideal point and zero point are chosen as the reference points. Suppose \( F_1 = \{ f_1, f_2, ..., f_n \} \) and \( G_1 = \{ g_1, g_2, ..., g_n \} \), where \( f_j \) and \( g_j \) are the positive ideal point and zero point of criterion \( q_j \). \( f_j \) and \( g_j \) are calculated by the following formula:

\[
\begin{align*}
    f_j &= \max \{ r_{ij} \mid i = 1,2,\ldots,m \} \\
    g_j &= \min \{ r_{ij} \mid i = 1,2,\ldots,m \} \quad (6)
\end{align*}
\]

(3) The determination of value function

According to the prospect value function given by formula (2), the value function of every criterion is obtained as follows:

\[
V(r_{ij}) = \begin{cases} 
(r_j - g_j)\alpha, & \text{if } g_j \text{ is the zero point} \\
-(\theta(f_j - r_j))\beta, & \text{if } f_j \text{ is the positive ideal point}
\end{cases} \quad (7)
\]

Where \( i=1, 2, ..., m \), \( j=1, 2, ..., n \).

(4) The determination of decision-making weight

Entropy was firstly introduced in information theory by C. E. Shannon, and now it is widely applied in engineering technology and social economy (Guo, 2001). The basic idea of the entropy method is to determine the weight according to the size of the index variation.

Generally speaking, the smaller the information entropy \( E_j \) of some index, the bigger the variation degree of the index value will be, and thus, the greater the effect of the index will be in a comprehensive assessment, therefore, the greater the weight of the index will be. On the contrary, the bigger the information entropy \( E_j \) of some index, the smaller the variation degree of the index value will be, and thus, the smaller the effect of the index will be in a comprehensive assessment, therefore, the smaller the weight of the index will be.

In this paper, the entropy method is adopted to determine the weight of different criteria in the multi-criterion decision making problem. Considering the normalized matrix \( R=(r_{ij})_{m \times n} \), according to the definition of entropy in the information theory, the entropy of the \( j \)-th criterion is:

\[
E_j = -k \sum_{i=1}^{m} p_{ij} , (j = 1, 2, ..., n)
\]

Where \( p_i = r_i / \sum_{i=1}^{m} r_{ij} \), \( k=1/\ln m \), and suppose that when \( p_{ij}=0, p_{ij}\ln p_{ij}=0 \).

Suppose the decision making weight vector of project \( a_i \) when faced with gains and losses are \( w_i^+ = \{ w_i^+(w_1), w_i^+(w_2), ..., w_i^+(w_m) \} \) and \( w_i^- = \{ w_i^-(w_1), w_i^-(w_2), ..., w_i^-(w_m) \} \) respectively, where \( w_i^+(w_j) \) and \( w_i^-(w_j) \) represent the decision weight of project \( a_i \) to criterion \( q_j \) when faced with gains and losses respectively. They can be calculated by the following formula:

\[
w_i^\pm (w_j) = \frac{(w_j)^\gamma}{[(w_j)^\gamma + (1-w_j)^\gamma]^\beta} \quad (9)
\]
Where \(0<\gamma<1, 0<\delta<1\). As for the evaluation of parameters \(\gamma, \delta\), some innovative results have been obtained in different application background. For example: \(\gamma=0.61, \delta=0.69\) are obtained by kahneman et al by demonstration, and (Richard and Wu, 1999) think that \(\gamma=0.74, \delta=0.74\).

(5) Calculate the prospect value of project

According to the prospect value function given by formula (1), the prospect value \(V_{ai}\) of every project \(a_i\) are obtained:

\[
V_{ai} = \sum_{j=1}^{n} [V^+(r^j_a)w^+_j(w_j) + V^-(r^j_a)w^-_j(w_j)]
\]

Where \(i=1, ..., m\).

(6) The selection of projects

The order of the whole project set can be obtained according to the comprehensive prospect value of project \(a_i\), and thus, the best investment project are got.

4. THE RISK DECISION MAKING ANALYSIS OF INVESTMENT IN REAL ESTATE IN HAINAN PROVINCE

An investment company is planning to invest in the real estate projects in some cities in Hainan Province, and there are six alternative cities: Haikou\(a_1\), Sanya\(a_2\), Qionghai\(a_3\), Wanning\(a_4\), Wenchang\(a_5\), Danzhou\(a_6\). In the decision-making process, the decision maker mainly takes the investment environment, market growth potential and market equilibrium condition into consideration. According to the market prediction, the above three criteria can be divided into eight criteria, namely: GDP\(q_1\), urban population\(q_2\), urban per capita disposable income\(q_3\), the actual use of foreign capital\(q_4\), fixed-asset investment\(q_5\), sales\(q_6\), investment volume\(q_7\) and saleable area/floor space completed\(q_8\). All the criteria are efficient type criteria. According to the above decision making information, the city of the most investment value will be selected.

4.1 Analysis

The case is a classical risk multi-criterion decision making problem. In the traditional risk decision making, a rational decision making based on expected utility theory is a feasible method, however, this method does not take the people’s bounded rationality into consideration. In this paper, the decision-making method based on the prospect theory considers the people’s bounded rationality, and it is more in line with people’s actual decision making.

4.2 Decision making process

There are six alternative cities of investing in real estate projects in Hainan Province, whose data of the criteria are obtained from the Hainan Statistical Yearbook of 2016. Object values of each criterion of each city are shown in table 1.
Table 1 Objective values of each criterion of each city

<table>
<thead>
<tr>
<th></th>
<th>q₁</th>
<th>q₂</th>
<th>q₃</th>
<th>q₄</th>
<th>q₅</th>
<th>q₆</th>
<th>q₇</th>
<th>q₈</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₁</td>
<td>1161.965</td>
<td>171.68</td>
<td>24442</td>
<td>42680.9</td>
<td>1012.046</td>
<td>2967448</td>
<td>4563949</td>
<td>170.57</td>
</tr>
<tr>
<td>a₂</td>
<td>435.8202</td>
<td>53.94</td>
<td>23668</td>
<td>26072</td>
<td>705.967</td>
<td>2036073</td>
<td>4666452</td>
<td>58.9</td>
</tr>
<tr>
<td>a₃</td>
<td>200.497</td>
<td>23.18</td>
<td>17971</td>
<td>591</td>
<td>174.2472</td>
<td>509570</td>
<td>1173570</td>
<td>134.02</td>
</tr>
<tr>
<td>a₄</td>
<td>165.823</td>
<td>25.8</td>
<td>17510</td>
<td>14424.9</td>
<td>158.0638</td>
<td>550832</td>
<td>981717</td>
<td>250.96</td>
</tr>
<tr>
<td>a₅</td>
<td>169.627</td>
<td>27.95</td>
<td>18606</td>
<td>1383</td>
<td>160.0166</td>
<td>367459</td>
<td>933912</td>
<td>94.92</td>
</tr>
<tr>
<td>a₆</td>
<td>443.2522</td>
<td>50.12</td>
<td>17464</td>
<td>41553.3</td>
<td>196.796</td>
<td>154420</td>
<td>272831</td>
<td>31.31</td>
</tr>
</tbody>
</table>

According to formula (5) and the data in Table 1, the normalized decision making matrix can be calculated, and it is shown as follows:

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 0.977 & 0.634 \\
0.271 & 0.207 & 0.89 & 0.605 & 0.642 & 0.669 & 1 & 0.126 \\
0.035 & 0 & 0.073 & 0 & 0.019 & 0.126 & 0.205 & 0.468 \\
0 & 0.018 & 0.007 & 0.328 & 0 & 0.141 & 0.161 & 1 \\
0.004 & 0.032 & 0.164 & 0.017 & 0.002 & 0.076 & 0.15 & 0.29 \\
0.279 & 0.181 & 0 & 0.973 & 0.045 & 0 & 0 & 0 \\
\end{bmatrix}
\]  

(12)

Take \( \alpha = \beta = 0.88 \), \( \theta = 2.25 \), by formula (7), the value function is got as follows:

\[
V(r_j) = \begin{cases} 
(r_g - g_j)^{0.88}, & g_j \text{ is the zero point project} \\
-2.25(f_j - r_g)^{0.88}, & f_j \text{ is the positive ideal project} 
\end{cases}
\]

(13)

Thus, the negative prospect value matrix is:

\[
\begin{bmatrix}
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & -0.081 & -0.929 \\
-1.704 & -1.835 & -0.323 & -0.994 & -0.911 & -0.850 & 0 & -1.999 \\
\end{bmatrix}
\]  

(14)

The positive prospect value matrix is:

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 0.980 & 0.670 \\
0.316 & 0.250 & 0.902 & 0.643 & 0.677 & 0.702 & 0 & 0.162 \\
0 & 0 & 0.100 & 0 & 0.031 & 0.162 & 0.248 & 0.513 \\
0.052 & 0.029 & 0.013 & 0.375 & 0 & 0.178 & 0.201 & 1 \\
0.008 & 0.048 & 0.204 & 0.028 & 0.004 & 0.104 & 0.188 & 0.336 \\
0.325 & 0.222 & 0 & 0.976 & 0.065 & 0 & 0 & 0 \\
\end{bmatrix}
\]  

(15)

Based on the entropy method, the weight of different criteria can be calculated by the formula (8), namely: \( w = (0.1505, 0.1630, 0.1403, 0.0867, 0.1814, 0.1127, 0.0960, 0.0694) \).

Take \( \gamma = 0.61, \delta = 0.69 \), the decision-making vector of being in gains and the one of being in losses can be calculated by formula (9) (10) respectively, namely: \( w^+ = (0.2273, 0.2362, 0.2197, 0.1737, 0.2487, 0.1975, 0.1826, 0.1554) \); \( w^- = (0.2172, 0.2277, 0.2083, 0.1561, 0.2426, 0.1828, 0.1660, 0.1364) \).
Calculated by formula (11), the prospect values of different cities are obtained as follows: \( V_{a_1} = 1.4460, V_{a_2} = -0.7039, V_{a_3} = -2.9725, V_{a_4} = -3.097, V_{a_5} = -2.5834 \).

According to the prospect values, the ranking of the cities is: \( a_1 > a_2 > a_6 > a_4 > a_3 > a_5 \). Therefore, the most suitable city for investment is Haikou.

5. CONCLUSION

The risk multi-criterion decision making methods have wide applications in risk decision making process, and the decision-making method based on prospect theory is more in line with people’s actual decision making. In this paper, take the investment value analysis of six cities in Hainan Province as an example, first, the positive and negative prospect value matrix of different criteria are obtained based on the double reference points (zero point, positive ideal point), then, an entropy method is introduced to calculate the weight proportion of different criteria, and therefore, the decision making vector of being in gains and the one of being in losses are got respectively, finally, sort and pick over the city of the most investment value according to the comprehensive prospect value of every city. Compared with the decision-making method based on expected utility theory, this method can better reflect the decision maker’s bounded rationality and subjective psychology, and it is of favorable practical application values.

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6. REFERENCES


