Design of High Precision Linear Encoder based on Direction and Subdivision Algorithm

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Abstract

In order to improve the precision and efficient of the linear encoder, the direction and subdivision algorithm is proposed in this paper. This paper introduces the principle of the subdivision and the related technique on signal processing. The principle of subdivision is improved amplitude subdivision technique, which would reduce the errors of traditional amplitude subdivision technique when approximate method is used. Besides, in the way of counting the zeros of the output sinusoidal waves and cosine waves, the circuit could be simplified and the accumulated errors caused by the circuit could thus be reduced. The experiment results of this paper have a reference value for the application of direction and subdivision algorithm in the design process of high precision linear encoder which can also promote the overall performance substantially.

Keywords: Evaluation standard, high precision, linear encoder, direction and subdivision algorithm

1. INTRODUCTION

Grating scale is a kind of precise detecting sensor and is widely used in modern precision measurement fields. For example, it is always used in angular measurement and linear measurement on CNC machine system. For its excellent characteristics of high precision, wide scale and fast response, it has been commonly used in the detection of closed-loop control of CNC machine system, especially in high-speed and high-precision processing activities. However, the test device for grating scale data available in the market requires so much professional knowledge to operate that the industrial workers cannot handle it cleverly and skillfully. Besides, the existing equipment is much more expensive, not suitable for small and medium enterprises. According to the above reasons, designing a low-cost, easy-to-operate grating scale test platform is becoming more and more practically significant. Rapid development of high-tech manufacturing leads to increasing demand for high accuracy fabrication of Computer numerical control (CNC) machine. The tools for linear displacement detection and control are changing from angle encoders with ball screws to linear encoders with linear shaft motors. However, hitherto no domestic linear encoder can meet the criteria of closed-loop CNC machine. The absolute linear encoder is a new type of linear encoder product launched in 2005. With the developments in the fields of integrated circuit (IC) packaging technology, ultra-precision machining, precision instruments and bio-medical engineering, the equipment related have increasingly demanding both on speed and positioning accuracy, so high-speed high-precision positioning technology play a crucial role in modern science and technology. As a crucial tool in Nano-displacement measurement, grating scale has the advantages of high-precision, anti-interference capability and long life, but due to the restrictions in the structure of optical signal detection circuit and scanning frequency, the maximum allowable speed of grating scale displacement measurement is inversely proportional to its velocity. For the motion characteristics in high-speed high-precision positioning system and the issue of high-speed displacement measurement of grating scale is difficult to achieve high resolution, this paper proposed a new measuring displacement method which is suitable for high-speed systems and can achieve high resolution at the end of the displacement based on single grating scale. Compared with incremental linear encoder, the absolute linear encoder has significant improvement in both function and performance. It has been widely used in the medium to high-end CNC system. Therefore, research on measuring technique of the high accuracy absolute linear encoder is of great importance. This paper studies the working principle of absolute linear encoder and proposes the research scheme, based on the domestic and foreign references. By intensive study on key technologies including the single track absolute
position coding, dedicated photoelectric device and serial communication protocol, the research on absolute linear encoder is finished. This paper is based on linear 20µm analogue signal period of open non-contact optical encoder produced by Renishaw, and adopted the circuit for the input signal pre-processing, the software in Labview, associating with the DAQ card by NI (National Instrument), to develop the technology on digital subdivision of linear optical encoder for Nano positioning within a relatively large-scale. This paper introduces the principle of the subdivision and the related technique on signal processing. The principle of subdivision is improved amplitude subdivision technique, which would reduce the errors of traditional amplitude subdivision technique when approximate method is used. Besides, in the way of counting the zeros of the output sinusoidal waves and cosine waves, the circuit could be simplified and the accumulated errors caused by the circuit could thus be reduced. The HP5529A laser interferometer is used to calibrate the effect of the subdivision within both large-scale and short-scale. In this paper, the subdivision error could be limited in 0.15 µm in large-scale while in short-scale it can be reduced to 20 nm and the resolution is 1.22 nm. Nano positioning can be realizable when this subdivision technique is applied.

2.MATERIALS AND METHODS

2.1 Overview

As a kind of sensor of displacement or angle, grating has been widely used in modern industry and national defence and other domains. The development of technology and increasing of practical application demanding have put forward higher requirement for measurement accuracy of grating measuring system (Venkata et al., 2015). Electronics subdivision technology of grating Moire Fringe signal has been a kind of effective method to improve measurement accuracy. By researching the principle of grating displacement measuring and subdivision technology for photoelectric Moire Fringe signal, a new subdividing method and system design scheme was proposed in this paper. This paper analyses the differential amplifier and phase shift circuit for analogy signal and the AD converter circuit and the digital circuit based on FPGA. It mainly analyses the principle of Coordinate Rotation algorithm which is used as method of nonlinear subdivision for 1/8period of Moire Fringe signal (Zou et al., 2015). This paper uses VHDLto describe the CORDIC algorithm for phase decoding and has realized it. This is an effective way to avoid the subdividing error which is caused by the traditional linear-fitting process of arc tangent function and keep the high times and nonlinear subdivision data valid in the process of displacement calculation. Meanwhile, the IEEE-754format single-precision floating-point calculation is applied in subdivision which is more accurate then fixedpoint calculation. This paper has designed and realized the hardware platform based on FPGA for the subdividing technology (Sougalo et al., 2011). The designed hardware system utilizes the parallel pipeline technology, so that it can simultaneously implement the function of counting, direction-judgement, driving the AD9826to complete data acquisition, decoding then linear-fitting phase with CORDIC algorithm, floating-point calculation for displacement, BCD decoding of float-point data, driving the LCD to display the measurement result real time and serial communication on a piece of FPGA. Finally, this paper gives an experimental verification for the measuring accuracy of the hardware circuit system, and then analyses the experimental result and the error factors which influences the measuring accuracy.

Precise positioning devise (Ko et al., 2014; Chicca et al., 2014; Devi et al., 2015) is more decisive in the field of nanotechnology in recent years when the electronics and semiconductor industry is developing rapidly. In this field, with the development of the process technique, smaller line width and more precise process even scale down to several nanometres has been developed constantly, which requires the technique of positioning and measurement should be improved accordingly. In modern industry, the typical precise measurement and positioning instruments applied on large-scale; besides the laser interferometer that is high precision scale measurement instrument can be classified into linear and rotary encoder. Because of the virtue of high precision, linear optical encoders have been applied widely. In order to get Nano positioning, it appears many researches of digital subdivision technology of linear optical encoder.

This study is based on linear 20µm analogue signal period of open non-contact optical encoder optical encoder produced by Renishaw(shown in Figure 1), and adopted the circuit for the input signal pre-processing and the software in Labview, associating with the DAQ card by NI (National Instrument), with which the study on digital subdivision of linear optical encoder for Nano positioning within a relatively large-scale has been carried out. The subdivision method mainly including phase-shifted interpolation method, digital interpolation method, triangle time’s frequency method, locked-phase subdivision method, modulates subdivision method, and amplitude subdivision method (Yuan et al., 2014; Peng, 2015) etc. For the former four methods, the complexity of circuit structure, usually with higher developing cost, may increase the accumulated error, and the circuit structures are more likely to be disturbed. The principle of the amplitude subdivision is to process the output
with phase-shifted sinusoidal and cosine signals. These two phase-shifted signals can be transformed into two square waves. In this way the grating period can be quadrupled and the direction of the optical encoder’s movement can be detected and the displacement of large-scale can be measured by counting both the falling and rising edges of the square waves. A triangle wave is available to the calculation of \( \text{abs} \left( \sin \theta - \cos \theta \right) \), which can be realized by a simple circuit. A straight subdivision can be used in each linear unit of triangle wave. This subdivision will cause non-linear error, which might be as large as 60nm, because the triangle wave we use is not an exact one but still a sinusoidal function wave. The subdivision in this study is based on the traditional amplitude subdivision but has some improvement, which has been testified to be able to upgrade the precision.

![Figure 1. Renishaw linear optical encoder](image1.png)

The principle of subdivision is improved amplitude subdivision technique, which would reduce the errors of traditional amplitude subdivision technique when approximate method is used (Lee et al., 2015; Leidner, 1999). Besides, in the way of counting the zeros of the output sinusoidal waves and cosine waves, the circuit could be simplified and the accumulated errors caused by the circuit could thus be reduced. Figure 2 shows the probe and data processing unit of Renishaw linear optical encoder.

![Figure 2. The probe and data processing unit of Renishaw linear optical encoder](image2.png)

### 2.2 The direction and subdivision algorithm

The output sinusoidal and cosine signals of the optical encoder, with amplitude of about only 150mV and DC offset of about 1.26V. It should be pre-processed before they enter the DAQ card. These pre-process circuits include amplification of the amplitude and elimination the DC offset (Shang et al., 2013). The principle of the circuit is shown in Figure 3.

Set \( \omega_s = Y \), \( \omega_c = X \), then the basic equation of the proposed equation is shown in equation (1):

\[
X = 2\pi k + \pi - \alpha - \pi M \sin Y \quad (1)
\]
\[ u_{p1}(X, Y) = \begin{cases} 
0 & \text{if } X < 2\pi k + \pi \alpha_1 - \pi M \sin Y \\
E/6 & \text{if } X \geq 2\pi k + \pi \alpha_1 + \pi M \sin Y
\end{cases} \] (2)

The double Fourier series of function \( \mu_{p1}(X, Y) \) is given:

\[ u_{p1}(X, Y) = \frac{A_{m}}{2} + \sum_{n=1}^{\infty} (A_{on} \cos nX + B_{on} \sin nY) + \sum_{n=1}^{\infty} (A_{nm} \cos mX + B_{nm} \sin mY) \] (3)

In the above formula

\[ A_{mn} + jB_{mn} = \frac{2}{(2\pi)^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} u_{p1}(X, Y)e^{j(mX+nY)}dXdY \] (4)

Take the formula (2) into formula (4)

\[ A_{mn} + jB_{mn} = \frac{E}{6\pi^2} \int_{0}^{\pi} \int_{2\pi k + \pi \alpha_1 + \pi M \sin Y}^{2\pi k + \pi \alpha_1 - \pi M \sin Y} e^{j(mX+nY)}dXdY \]

\[ = \frac{E}{jm\pi} e^{jm(\pi-\alpha_1)} \left[ \frac{1}{\pi} \int_{0}^{\pi} e^{jnm\pi \sin Y} e^{jnY} dY \right] \] (5)

By Bessel function,

\[ \frac{1}{\pi} \int_{0}^{\pi} e^{jnm\pi \sin Y} e^{jnY} dY = J_{m}(mM \pi) \frac{e^{jm\pi} - 1}{2} \] (6)

\[ \frac{1}{\pi} \int_{0}^{\pi} e^{-jnm\pi \sin Y} e^{jnY} dY = J_{m}(mM \pi) \frac{1 - e^{jm\pi}}{2} \] (7)

Then,
\[ A_{mn} + jB_{mn} = \frac{E}{j\omega n} e^{j\omega (x-n)} \]

\[ [J_n(m\pi)\frac{e^{j\omega x} - 1}{2} - J_n(m\pi)\frac{1 - e^{j\omega x}}{2}] \]

\[ = j\frac{E}{6\omega n} J_n(m\pi)e^{j\omega (x-\omega)}[1 - e^{j\omega x}] \]

The linear equation can be expressed into the following simplified forms:

\[ L(\nabla, \omega)f(x, \omega) = 0 \]

\[ L(\nabla, \omega) = T(\nabla) + \omega^2 \rho J \]

In which,

\[ T(\nabla) = \begin{bmatrix} T_{ik} & t_i(\nabla) \\ t_k^T(\nabla) & -\tau(\nabla) \end{bmatrix}, \quad J = \begin{bmatrix} \delta_{ik} & 0 \\ 0 & 0 \end{bmatrix}, \quad f(x, \omega) = \begin{bmatrix} u_{ij}(x, \omega) \\ \varphi(x, \omega) \end{bmatrix} \]

Consider delay, the \( L \) can be expressed as:

\[ L^0 = \begin{bmatrix} C_{ijkl} \delta_{ik} & e_{ij}^0 \\ e_{ij}^T & -\eta_{ik}^0 \end{bmatrix} \]

These functions can be expressed in the following form:

\[ C(x) = C^0 + C^1(x), \quad e(x) = e^0 + e^1(x) \]

\[ \eta(x) = \eta^0 + \eta^1(x), \quad \rho(x) = \rho^0 + \rho^1(x) \]

The value with superscript of 1 represents the difference below:

\[ C^1 = C - C^0, \quad e^1 = e - e^0 \]

\[ \eta^1 = \eta - \eta^0, \quad \rho^1 = \rho - \rho^0 \]

And local fractional integral of \( f(x) \) defined by Eq. 6.

\[ a \int_b^{(\alpha)} f(t) = \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t)(dt)^\alpha \]

\[ = \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta \to 0} \sum_{j=0}^{j=N-1} f(t_j)(\Delta t_j)^\alpha \]
3. RESULTS AND DISCUSSION

The principle of the signal subdivision of this study is shown in Figure 4. The signals though the pre-processing circuit is steady and have the same amplitudes. These two signals with the phase difference of 90 degrees can be transformed into two phase-shifted square waves. And through a multiplication operation these two square waves will be transformed into another square wave with a doubled frequency. In this way the sinusoidal and cosine wave is quadrupled. Each linear unit, a rising or a falling segment of the triangle wave ($|\sin \theta| - |\cos \theta|$) represent a quarter of one grating period (5 µm). All the mathematical operation is upon the data of voltage acquired DAQ card and the circuit structure is thus simplified and the error caused by the circuit is reduced. The condition of the signal is different in the movement directions of optical encoder. It is easy to find that when the optical encoder move forward, the high level of the double-frequencies square wave is corresponding to the falling segment of the triangle wave and the condition reverses when the optical encoder move backward. So, the direction of the optical encoder’s movement can be detected. In most of the situation we control, the direction of the movement actively. For the real Nano positioning, the subdivision is still needed. A typical method is to regard the $|\sin \theta - \cos \theta|$ wave as a triangle wave, upon which the mathematical operation is carried out more simple. But this method will inevitably introduce non-linear errors. In this study, by calculating the values of the function $|\sin \theta - \cos \theta|$ straight. The situation of signs here is somewhat complex so this paper take $|\sin \theta - \cos \theta|$ for instance. Let the value of voltage is $y$, then,

\[
\frac{1}{\Gamma(1+\alpha)} \int_{x}^{\infty} \frac{f(t)}{(t-x)^{\alpha}(dt)^{\alpha}} = \lim_{\varepsilon \to 0} \left[ \frac{1}{\Gamma(1+\alpha)} \int_{x-\varepsilon}^{\infty} \frac{f(t)}{(t-x)^{\alpha}(dt)^{\alpha}} \right] + (15)
\]

\[
\frac{1}{\Gamma(1+\alpha)} \int_{x}^{\infty} \frac{f(t)}{(t-x)^{\alpha}(dt)^{\alpha}} = \frac{1}{Z(x)} \exp \left( \sum_{i=1}^{K} \lambda_i f_i(x, y) \right). \quad (16)
\]

\[
Z(x) = \sum_{i=1}^{K} \exp \left( \sum_{i=1}^{K} \lambda_i f_i(x, y) \right). \quad (17)
\]

The sum of characteristic function is
\[
\sum_{i=1}^{K} f_i(x, y) = C, \quad \lambda_i^{(0)} = 1(18)
\]

\[
\lambda_i^{(n+1)} = \lambda_i^{(n)} \left[ \frac{E_p f_j}{E_p^{(n)} f_j} \right]^{1/2} . \quad (19)
\]

The step is shown as the following:

1. Each destination node broadcasts a HELLO Message. Each node which receives the HELLO message uses it in order to establish its cost with respect to this destination node.

2. Each node then broadcasts its ID, the ED of the destination node as well as its cost C with respect to the corresponding destination node. A node (that hears the message) will replace its previous cost C_p with respect to a given destination node only if the new computed cost C+1 is smaller than C_p.

3. At the end of this phase, each node will have a cost with respect to each of the destination nodes of the network but also the ID and costs of its neighbours. The second phase involves the route discovery agents and corresponds to the route discovery phase.

4. When a node needs to send data towards a given destination node, it will send route discovery agents in order to find routes towards the corresponding destination node.

5. It will generate a random number between 0 and 1.

   (1) If this number is between a parameter P and 1, then it will broadcast a message towards a neighbour which has a higher cost than its own cost which means practically that the message will be broadcasted towards a node which is closer to the destination node.

   (2) If this number is between 0 and another parameter a, then it will broadcast the message towards a neighbour which has the same cost as its cost.

   (3) If this number is between the parameters a and p, then it will broadcast the message towards a neighbour with a smaller cost than its cost.

Equation (3) completely eliminates the non-linear error. The Labview software supplies a ready-made function ‘arcsine’ that can be used conveniently and we don’t need to find any approximate operation. The DAQ card gets the voltage value of sinusoidal and cosine signals, and then the mathematical operation should be done. With the equation (3) the displacement x1 can be calculated. After that, through some logic judgments we can decide whether x1 or (5-x1) should add to the total displacement x. The DAQ card acquires voltage values of the sinusoidal and cosine signals constantly, reserve them in the buffers and then the count of zeros is accumulated (this affection is equal to counting the edges of the double-frequencies square wave). If one zero is detected it means 5 µm has passed. The beginning part and the ending part of the displacement can be calculated with the equation (3) and some logic judgments while the middle part can be calculated with the number of the zeros. Then the total displacement includes these three parts, shown in figure 4. The DAQ card used here has 12-digit A/D converter and the limitation of the input signal can be adjusted so one of the single rising or falling segment can be 4096 subdivisions. so the resolution can reach to 1.22 nm.
Meanwhile, the IEEE-754 format single-precision floating-point calculation is applied in subdivision which is more accurate than fixed-point calculation. This paper has designed and realized the hardware platform based on FPGA for the subdividing technology. The designed hardware system utilizes the parallel pipeline technology, so that it can simultaneously implement the function of counting, direction-judgement, driving the AD9826 to complete data acquisition, decoding the nonlinear fit phase with CORDIC algorithm, floating-point calculation for displacement, BCD decoding of float-point data. In order to evaluate the capability and the precision of the total subdivision system, in this study a PCLM (Piezo Ceramic Linear Motor by Anorad Company) stage, including a linear optical encoder and a HP5529A laser interferometer is used to calibrate the subdivision results. The principle of the experiment is shown in Figure 5, the experimental setup is shown in Figure 6, and the result of the calibration is shown in Figure 7.

Figure 4. The principle of the subdivision

Figure 5. The subdivision the principle of the experiment
In order to evaluate the capability and the precision of the total subdivision system in a short-scale, the setup mostly like the system introduced above is used, and the static driving mode (one mode of the piezoelectric ceramic motor) of the Nano motor is applied, so a short-scale within ±400 nm can be acquired with the Nanomotor. The experiment was repeated for 5 times to achieve the average value of the results. The result of the subdivision and the datum of the HP5529A laser interferometer has been detected as shown in Fig. 8. The above graphs show that the subdivision of linear optical encoder in this study can limit the errors in 0.15 µm when used in large-scale. In short-scale within ±400 nm the error of the subdivision is 20 nm and the resolution is 1.22 nm. The subdivision in large and short-scale introduced above can be used in feedback control of the stage to realize Nano positioning.
Figure 8. The calibration results in short-scale

4. CONCLUSION

As a kind of sensor of displacement or angle, grating has been widely used in modern industry and national defence and other domains. The development of technology and increasing of practical application demanding have put forward higher requirement for measurement accuracy of grating measuring system. In order to improve the precision and efficient of the linear encoder, the direction and subdivision algorithm is proposed in this paper. This paper introduces the principle of the subdivision and the related technique on signal processing. The principle of subdivision is improved amplitude subdivision technique, which would reduce the errors of traditional amplitude subdivision technique when approximate method is used. This paper has designed and realized the hardware platform based on FPGA for the subdividing technology. The designed hardware system utilizes the parallel pipeline technology, so that it can simultaneously implement the function of counting, direction-judgement, driving the AD9826to complete data acquisition, decoding thenonlinearfit phase with CORDIC algorithm, floating-point calculation for displacement, BCDdecoding of float-point data, driving the LCD to display the measurement result real time and serial communication on a piece of FPGA. Besides, in the way of counting the zeros of the output sinusoidal waves and cosine waves, the circuit could be simplified and the accumulated errors caused by the circuit could thus be reduced. In short-scale within ±400 nm the error of the subdivision is 20 nm and the resolution is 1.22 nm. The subdivision in large and short-scale introduced above can be used in feedback control of the stage to promote the overall performance substantially.

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