Coordination of supply chain with a \((s, S)\) inventory ordering policy under random yield and demand

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Abstract

The main purpose of this paper is to investigate the coordination of a supply chain consisting of a raw material supplier and an end product manufacturer who orders products from the supplier who suffers from random yield to meet a stochastic demand in a single-period selling season, where the order quantity satisfies \((s, S)\) inventory policy. We first determine the optimal \((s, S)\) ordering policy and the corresponding raw material production quantity that maximize expected profit of the centralized supply chain, and find that our \((s, S)\) ordering policy can significantly enhance the supply chain’s expected profit compared to an invariable ordering policy. Then we analyze the decentralized scenario and present that the commonly used wholesale price contracts cannot coordinate the system. We further propose a contract combining buy-back and shortage penalty (BBSP) to achieve supply chain coordination. Finally, the numerical examples provide strong evidence for the view that the profit increase becomes more obvious as the supply uncertainty or the demand uncertainty is more severe in the \((s, S)\) inventory ordering policy compared to an invariable ordering policy and the BBSP contract can effectively improve the performance of the whole supply chain.

Keywords: coordination, uncertainty, BBSP contract, \((s, S)\) inventory ordering policy

1. INTRODUCTION

With the globalization of markets and competition, supply chain management becomes very popular and important. The expanding variety in customer demand poses challenging tasks for supply chains in dealing with that variety. Most manufacturers not only have to deal with demand uncertainty but supplier uncertainty as well. Because of long lead times, complicated production processes, and unpredictable factors like weather and environment, the yield of a raw-material manufacturer is often lower than the initial production quantity. In fact, in recent years, there has been a lot of emphasis on models with supply uncertainty as well. These include yield loss (related to quality considerations), unreliable machinery and unplanned maintenance. We refer interest readers to Grosfeld-Nir and Gerchak (2004) and Yano and Lee (1995) for summaries of the effects of random supply on inventory control.

Since Arrow et al. (1951) put forward the famous \((s, S)\) inventory control strategy, most of the literatures indicate that the \((s, S)\) inventory ordering strategy is a suitable for all kinds of situation and effective inventory control strategy, especially in the case of the random demand. This article introduces the \((s, S)\) inventory ordering strategy into supply chain system to consider coordination problem in the case of both supply and demand uncertainty. To the best of our knowledge, there has not been any published paper that has addressed such issue so far. Hence, our study sheds new light on the management of the supply chain with \((s, S)\) inventory ordering under random supply and stochastic demand.

A contract mechanism is an effective method for achieving coordination within the supply chain, which provides incentives to share risks and/or rewards by modifying the terms of trade through the introduction of trade parameters between the entities. Although the supply chain coordination is not a new problem, it is rare to simultaneously consider stochastic supply and random demand in the existing literatures. In this paper we study...
the supply chain coordination issues associated with demand uncertainty and supply uncertainty. In our model, the manufacturer and the supplier first propose a (s, S) inventory ordering contract for raw materials. The manufacturer determines (s, S) inventory ordering quantity before the demand is known, and the supplier is responsible for determining raw-material production quantity before the production season. The manufacturer, which faces a random yield supply, will purchase raw materials from the supplier at an amount that meets the random market demand. In the case that the supplier does not yield enough raw materials to satisfy the manufacturer, it will purchase the difference from a spot market. The manufacturer then produces the amount as the supplier’s delivery quantity and the products are sold to the retailer. Finally, demand is observed, revenues are collected, and profits are earned.

He and Zhao (2012) investigate the ordering policy of the retailer, raw-material planning decision of the supplier, and the optimal contracts for a three level supply chain with random yield and demand. Our study is most related to He and Zhao (2012), but with several major differences. The differences are as follows: (1) He and Zhao (2012) study three-echelon supply chain and assume that the manufacturer meet a deterministic demand of the retailer, while we treat the manufacturer and the retailer as an entity (called manufacturer) for simplifying the expression and we assume that the manufacturer directly face the random demand. (2) He and Zhao (2012) investigate the retailer’s optimal invariable ordering quantity policy. Here we extend He and Zhao’s model by adopting a (s, S) inventory ordering policy. We find that a (s, S) inventory ordering policy can reduce the risk of yield uncertainty, and significantly enhance the profit of the supply chain. (3) For supply chain coordination, we develop a new contract combining buy-back and shortage penalty (BBSP) to achieve supply chain coordination. Under the shortage penalty contract in this paper, the manufacture sets up a quantity target and the supplier will accept a shortage penalty if the supplier’s delivery quantity is below the target.

Our major contributions to the literature are as follows. First, we develop a single-period inventory model with a (s, S) inventory ordering policy under yield and demand uncertainty, and characterize the production decision of the supplier and the (s, S) inventory ordering decision of the manufacture. Second, we investigate the efficiencies of (s, S) inventory ordering decision compared to the traditional invariable ordering policy and make the result of He and Zhao (2012) to be our special cases. Third, we propose a new shortage penalty contract, and show that a buy-back contract with shortage penalty can not only coordinate the supply chain but achieve arbitrary allocation of the optimal supply chain profit within a certain range between the manufacturer and the supplier by adjusting contract parameters.

The rest of this paper is organized as follows. In Section 2, we discuss related literature and then in Section 3, we introduce a centralized model and analyze the benchmark scenario. In Section 4, both the manufacture’s decision and the supplier’s decision are given in the decentralized scenario. A BBSP contract is analyzed for the supply chain coordination in Section 5 and present numerical examples in Section 6. Finally, conclusions and suggestions for future research are given in Section 7.

2. LITERATURE REVIEW

There are three research areas that are most relevant to our study, namely random yield, (s, S) inventory models, and supply chain contracts. Supply contracts have been a hot research topic in supply chain management. There is a considerable body of literature in this area. In the following we review the studies that are most relevant to our study.

Without a properly designed contract, there is the problem of double marginalization because the downstream retailer does not have an incentive to order enough inventories to maximize the total supply-chain profit (Spengler, 1950). Lariviere and Porteus (2001) reveal that the simple wholesale price contract is unable to coordinate the two-level supply chain. Pasternack (2008) shows that a returns policy, when the parameters are properly chosen, can coordinate the supply chain. Gerchak and Wang (2004) analyze the performance of revenue-sharing contract comparing with wholesale price contract in assembly systems with random demand. Cachon (2003) provides a detailed review of this literature.

Because the simple wholesale-price-only contract cannot coordinate the supply chain, researchers have been focusing on designs of contracts to coordinate a supply chain, such as revenue sharing contracts (Cachon and Lariviere, 2005; Van Der Rhee et al., 2010; Chakraborty et al., 2015), returns policies or buy-back contracts (Pasternack, 2008; Emmons and Gilbert, 1998; Ding and Chen, 2008; Wang and Choi, 2014), risk sharing contracts (Li and Kouvelis, 1999; He and Zhang, 2008; Inderfurth and Clemens, 2014), quantity flexibility contracts (Tsay, 1999; Lian and Deshmukh, 2009; Mahajan, 2014), quantity discount policies (Corbett and De
Groote, 2000; Li and Liu, 2006; Zissis et al., 2015), sales rebate contracts (Taylor, 2002; Chiu et al., 2012; Wong et al., 2009), and so on. The buy-back contract is widely used in industries with short life-cycle products such as books, personal computers, toys, etc. It is known that the returns policy can eliminate the double marginalization in a decentralized supply chain and enhance supply chain performance (Hu et al., 2013). Therefore, returns policy is one of the most commonly studied coordination contracts in literature, under many complicated situations. He et al. (2009) consider a condition in which the stochastic market demand is sensitive to both retail price and sales effort. They show that coordination is achieved by using a properly designed returns policy with a sales rebate and penalty contract. Xiong et al. (2011) introduce a composite contract for a two-stage supply chain by organically combining two component contracts: a buy-back contract and a quantity flexibility contract. They show that the composite contract has advantages over both component contracts in terms of supply chain coordination, profit allocation, and risk allocation. Chen and Bell (2011) investigate a channel that consists of a manufacturer and a retailer where the retailer simultaneously determines the retail price and order quantity while experiencing customer returns and price dependent stochastic demand. They propose an agreement that includes two buy-back prices, one for unsold inventory and one for customer returns, and show that this revised returns policy can achieve perfect supply-chain coordination and lead to a win-win situation. Liu et al. (2012) study the buy-back contract in perishable supply chain with delivery delay risk, and find that the coordination buy-back price in delivery delay is less than that in non-delivery-delay. For a more detailed discussion about returns policies, the reader is referred to Gurnani et al. (2010) and the references therein. So far buy-back contracts have been applied mainly to coordinate supply chains with only demand uncertainty. In this paper, we implement this returns policy combining shortage penalty to coordinate a supply chain with both supply and demand uncertainty.

We now briefly review papers devoted to (s, S) inventory models. Classical papers on the optimality of (s, S) policies in dynamic inventory models with stochastic demands and fixed setup costs are those of Arrow et al. (1951) and Dvoretzky et al. (1953). Kumar (1992) has attempted to extend the classical inventory model by incorporating service level and storage capacity constraints, but without a rigorous proof. Sethi and Cheng (1997) demonstrate the optimality of (s, S) policies in inventory models with Markovian demand. Recently, Ko et al. (2016) consider an (s, S) perishable inventory model with impatient customers and random lead times. They find an efficient approximation method for the joint stationary distribution of the number of items in the system and the optimal values of s and S to minimize system costs. One of the most important developments in the inventory theory has been to show that (s, S) policies are optimal for a class of dynamic inventory models with random periodic demands and fixed ordering costs. Therefore, we will introduce the theory of (s, S) inventory into the supply chain management and demonstrate the optimality of (s, S) policies to improve the efficiency of the supply chain.

The last stream of related literature focuses on the production and replenishment decisions with random yield. This research area related to random yield, started by Karlin (1958), has received a lot of attention in operations research. Henig and Gerchak (1990) and Wang and Gerchak (1996) provide detailed reviews and discussions in this field. Although the supply chain coordination is not a new problem, it is rare to simultaneously consider random yield and random demand in the existing literatures. Gurnani and Gerchak (2007) study coordination of a decentralized assembly system in which the demand of the assembler is deterministic and the component yields are random. They present incentive alignment control mechanisms under which system coordination is achieved. Yan et al. (2010) extend Gurnani and Gerchak’s model to the case of positive salvage value and n asymmetric suppliers, and show that the shortage penalty contract which can coordinate Gurnani and Gerchak’s model no longer coordinates the extended model. Güler and Bilgic (2009) consider a decentralized assembly system in which the customer demand and the yield of the suppliers are random. They establish the concavity of expected supply chain profit for arbitrary number of suppliers and propose two contracts and show that they coordinate the chain under forced compliance. Li et al. (2013) examine the dynamic pricing and supply chain coordination in a decentralized system that consists of one supplier and one manufacturer, in which both the market demand and production yield are stochastic. They propose a reimbursement contract to coordinate the decentralized supply chain so as to achieve the maximized profit. Güler and Keski (2013) analyze coordination in a supply chain with random yield and random demand. They study wholesale price, buy-back, revenue share, quantity flexibility and quantity discount contracts. They obtain that the randomness in the yield does not change the coordination ability of the contracts but affects the values of the contract parameters. Luo and Chen (2015) investigate the coordination of a supply chain consisting of a loss-averse supplier and a risk-neutral buyer who orders products from the supplier who suffers from random yield to meet a deterministic demand. They provide explicit conditions on which the random yield supply chain with a loss-averse supplier can be coordinated under shortage-penalty-surplus-subsidy contracts. Hosoda et al. (2015) investigate the impact of advance notice of product returns on the performance of a decentralised closed loop supply chain. They demonstrate that lead times, random yields and the parameters describing the returns play a significant role in
the benefit of the advance notice scheme. Hu et al. (2013) study a flexible ordering policy among a manufacturer and a supplier with random yield and demand uncertainty. They propose a revenue sharing policy with an order penalty and rebate (OPR) contract to fully coordinate the supply chain. Our study in this paper is similar to the works by Hu et al. (2013), but with at least two major differences. First, Hu et al. (2013) study a flexible ordering policy, while we adopt a (s, S) inventory ordering policy. Furthermore, they propose a revenue sharing policy with an order penalty and rebate contract to coordinate the supply chain, while we apply a contract combining buy-back and shortage penalty to achieve supply chain coordination.

3. PROBLEM DESCRIPTIONS AND THE INTEGRATED BENCHMARK

3.1 Problem Descriptions

Consider a supply chain consisting of a raw material supplier and an end product manufacturer who orders products from the supplier who suffers from random yield to meet a stochastic demand in a single-period selling season. The supplier and the manufacturer are both risk neutral and maximize their expected profit. The raw material supplier is subject to a random yield risk in the production process. When the planned production quantity of the supplier is $Q$, then the yield will be $\epsilon Q$, where $\epsilon$ is a nonnegative random variable with support on $[a, b]$ and $0 \leq a \leq b \leq 1$, and is characterized by cumulative distribution function (CDF), $G(y)$, and probability density function (PDF), $g(y)$, and $E(\epsilon) = \mu_1$. We assume that the production yield of the manufacturer is deterministic. These two assumptions could be justified by the many uncertainties (e.g., weather, environment, availability) involved in the raw material production process and the relatively stable environment (i.e., assembly) of the production of finished products. Suppose that the manufacturer faces a stochastic demand, $X$, with CDF, $F(x)$, PDF, $f(x)$, and $E(X) = \mu_2$. We also assume that both PDFs are strictly positive and both CDFs are non-negative and strictly increasing. $\epsilon$ and $X$ are continuous, differentiable, invertible and independent random variables, respectively.

The sequence of events is as follows:

The manufacturer determines the order quantity and orders the raw materials from the supplier (we assume that one product needs one raw material, but the results of this paper will not be affected by relaxing this assumption);

The supplier determines the production quantity, $Q$, for the raw materials and produces the raw materials. The supplier’s production cost is $c$ per unit. Since the supplier is uncertain, if the supplier can not produce the required quantity, it will buy the gap from the spot market at price $c$, which is exogenous;

The supplier charges the manufacturer a wholesale price $\omega$ per delivery unit. The manufacturer receives the raw materials and produces the products. The manufacturer’s production cost (i.e., assembly cost) is $c_m$;

The manufacturer sells the finished products to the customer at price $p$ per item;

The demand is realized.

To simplify the analysis, we assume that there is no setup cost for both the supplier and manufacturer, and unsatisfied demand will be lost for the manufacturer. We make the following assumptions to avoid uninteresting cases.

Assumption 1. $w > c_1$, $p > w + c_m$ and $p > c + c_m$.

Assumption 2. $p > c_1 / \mu_1, c_m$.

Assumption 3. $c, c_2 / \mu_1$.

Assumption 1 makes sure that the manufacturer and the supplier are willing to participate. Assumption 2 indicates that the manufacturer’s selling price is higher than the expected supply chain cost. Assumption 3 avoids
the case that the supplier never produces. Throughout this paper, we denote $\tilde{F}(x) = 1 - F(x)$, $x^* = \max(x, 0)$ and let $I(A)$ be the indicator function of event $A$.

### 3.2 (s, S) inventory ordering policy

The manufacturer and the supplier negotiate a (s, S) inventory ordering quantity, where S is such that the supplier’s delivery quantity is raised to the order-up-to level $S$ through buying the gap from the spot market if the supplier’s production decreases to a value equal to or smaller than the order level $s$. If the supplier’s production yield is more than minimum order quantity, $s$, the manufacturer is willing to accept the production random yield quantity $\epsilon Q$.

We assume the following sequence of events. First, the manufacturer decides the (s, S) inventory ordering policy. After receiving the manufacturer’s ordering decision, the supplier decides the production lot sizing, $Q$, and the production random yield $\epsilon Q$ is realized. In this stage, if the production output of the supplier is less than $s$, then the supplier will buy the gap from the spot market at price $c$. On the other hand, if the production output of the supplier is higher than $s$, then the buyer only choose to accept the production random yield quantity $\epsilon Q$.

Hence, after observing the production yield, the supplier’s delivery quantity, $D$, is given by

$$D = \begin{cases} S, & s \geq \epsilon Q, \\ \epsilon Q, & s < \epsilon Q. \end{cases}$$  \hspace{1cm} (1)

Next, the manufacturer receives the supplier’s delivery quantity $D$ and produces the products. Finally, the demand $X$ is realized and the manufacturer sells the amount to the customer.

### 3.3 The integrated benchmark

To establish a performance benchmark, we first analyze the optimization problem of an integrated supply chain.

In the centralized setting, there is a single decision maker whose aim is to maximize the expected profit of the chain which is the sum of the profits of the manufacture and the supplier. The integrated supply chain’s expected profit, denoted by $\Pi ms(s, S, Q)$, is defined as the sum of the expected profit function of the manufacturer and the supplier. Hence, by denoting $E[\min(y, X) = \int_0^x xf(x)dx + yF(y)]$, the expected supply chain profit can be expressed as

$$\Pi ms(s, S, Q) = E[p \min(D, X) - c(S - \epsilon Q)I(s \geq \epsilon Q) - c_m D] - cQ$$

$$= p\left\{ \int_{\min(S, X)}^{\epsilon Q} E[\min(y, X)]g(y)dy + \int_{\epsilon Q}^{\max(y, X)} g(y)dy \right\} -$$

$$c\int_{\min(S, X)}^{\epsilon Q} g(y)dy - c_m \left[ S \int_{\min(S, X)}^{\epsilon Q} g(y)dy + \int_{\epsilon Q}^{\max(y, X)} yg(y)dy \right] - cQ.$$

The first term in $\Pi ms(s, S, Q)$ is the total revenue from sales, the second term is the cost of buying raw materials from the spot market in the case of a low yield, and the third and fourth terms are costs for producing raw materials and finished products.

From the above equation, $\Pi ms(s, S, Q)$ can be rewritten as

$$\Pi ms(s, S, Q) = \int_{\min(S, X)}^{\epsilon Q} \left\{ pE[\min(S, X)] - (c + c_m)S \right\} g(y)dy$$

$$+ \int_{\epsilon Q}^{\max(y, X)} \left\{ pE[\min(yQ, X)] - (c_m + c)yQ \right\} g(y)dy$$

$$+ cQ\mu_t - cQ.$$  \hspace{1cm} (2)

Note that if $D \equiv S$, then the manufacturer’s ordering quantity is invariable and the objective function is given by
\[
\prod_r(S_i, Q_i) = E \left[ p \min(S_i, X) - c(S_i - \epsilon Q_i)^+ \right] - c_m S_i - c_i Q_i
\]
\[
= p \int_0^{S_i} xf(x)dx + S_i F(S_i) - c \int_0^{S_i/Q_i} (S_i - yQ_i) g(y)dy - c_m S_i - c_i Q_i.
\]

(3)

The above model is considered by He and Zhao (2012), therefore, the results of He and Zhao (2012) are special cases in our paper.

From Eq. (2), we can obtain the following optimal ordering decision making for the integrated supply chain.

**Proposition 1.** The supply chain’s expected profit \( \prod ms \) has the maximum value at \( (s^0, S^0, Q^0) = (s^0, F^{-1}\left((c + c_m)/p\right), Q^0) \) where \( s^0 \) and \( Q^0 \) satisfy the following equations

\[
\int_0^{s^0} xf(x)dx - \int_0^{s^0} xf(x)dx + s^0 F(s^0) = \left(\frac{c_m + c}{p}\right)s^0.
\]

(4)

\[
\int_{p/Q^0}^b \left[ pyF(yQ^0) - (c_m + c) y \right] g(y)dy = c_s - c \mu_i.
\]

(5)

All proofs, if not provided in the paper, are in the Appendix.

Substituting (4), (5) and \( s^0 \) into (2), we have the optimal supply chain’s profit as

\[
\prod_{ms}^0 (s^0, S^0, Q^0) = p \left\{ \int_0^{s^0/Q^0} \left( \int_0^{s^0} xf(x)dx \right) g(y)dy + \int_{p/Q^0}^b \left( \int_0^{s^0} xf(x)dx \right) g(y)dy \right\}.
\]

(6)

From Proposition 1, if \( D \equiv S \), then we have the following results.

**Corollary 1.** The supply chain’s expected profit \( \prod ms \) has the maximum value when the unique \( (S^0_i, Q^0_i) \) satisfies the following equations

\[
pF(S^0_i) - cG\left(\frac{S^0_i}{Q^0_i}\right) - c_m = 0, \quad (7)
\]

\[
c_{ij} \int_a^{s^0/Q^0_i} yf(y)dy - c_s = 0.
\]

(8)

See He and Zhao (2012).

Substituting Eqs. (7) and (8) into Eq. (3), we have \( \prod_{ms}^0 (S^0_i, Q^0_i) = p \int_0^{s^0} xf(x)dx. \)

**Proposition 2.** \( \prod_{ms}^0 (s^0, S^0, Q^0) > \prod_{ms}^0 (S^0_i, Q^0_i) \).

Proposition 2 shows that our \( (s, S) \) inventory ordering policy will enhance the integrated supply chain’s expected profit, since it makes full use of the supplier’s raw materials rather than by using an invariable ordering policy.

4. THE DECENTRALIZED SUPPLY CHAIN
In the decentralized setting, the supplier and the manufacture are independent agents and derive their individual expected profits. Each firm is risk neutral, so each firm maximizes its expected profit. There is full information, which means that both firms have the same information at the start of the game, i.e., each firm knows all costs, parameters and rules. We formulate the decentralized problem as a Stackelberg game, where the manufacturer is a leader and the supplier is a follower. Using backward induction, we first solve the manufacturer’s problem on \((s, S)\) inventory orders.

4.1 The manufacture’s problem

For any given \(Q > 0\), the expected profit function of the manufacture, denoted by \(\Pi ms (s, S)\), is

\[
\Pi_m(s, S) = E\left[p \min(D, X) - (w + c_m)D\right] \\
= p \int_0^{s/Q} E\left[\min(S, X)\right] g(y)dy + \int_{s/Q}^{S} E\left[\min(yQ, X)\right] g(y)dy - \\
(w + c_m) \left[S \int_0^{s/Q} g(y)dy + Q \int_{s/Q}^{s/Q} yg(y)dy\right],
\]

where the first term is the manufacturer’s revenue from the sales, and the second term is the replenishment costs for the raw material and the costs incurred by producing the end products.

From Eq. (9), using the similar proof as Proposition 1, the optimal \((s^*, S^*)\) that maximizes \(\Pi ms (s, S)\) can be obtained as follows:

\[
s^* = S^* = F^{-1}\left(\frac{w + c_m}{p}\right).
\]

The above equation implies that if there is no any incentive mechanism, the manufacturer’s optimal order quantity is an optimal invariable value \(s^*\). Substituting Eq. (10) into Eq. (9), we have the following manufacturer’s profit function.

\[
\Pi_m(s^*, S^*) = p \int_0^{s^*} x f(x)dx.
\]

4.2 The raw-material supplier’s problem

Following the sequence of events listed in Section 3 and using backward induction, next we solve the raw-material supplier’s production planning problem. After the manufacturer determines the invariable order quantity the expected profit of the supplier, denoted by \(\Pi s(Q)\), is

\[
\Pi_s(Q) = E\left[ws^* - c(s^* - sO)^* - c_Q\right] \\
= ws^* - c_Q - c \int_0^{s/Q} (s^* - yQ) f(y)dy.
\]

The first term of \(\Pi s(Q)\) is the supplier’s revenue from the manufacturer’s order. The second term is the raw-material production cost, and the last term is the cost associated with buying from the spot market if the raw-material production yield is low.

The next proposition is straightforward from Eq. (12).

Proposition 3. The raw-material supplier’s optimal production quantity \(Q^*\) is uniquely solved by

\[
\int_0^{s/Q} yg(y)dy = \frac{c_s}{c}.
\]

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Let $\eta = s^*/Q^*$, then from Eq. (13) we can find that $Q^*$ is a linear function on the manufacture’s ordered quantity. The linear coefficient depends on the production cost, the emergency purchasing cost from spot market, and the random yield distribution.

Substituting Eq. (13) into Eq. (12), then the supplier’s expected profit is

$$\Pi_s(Q^*) = [w - cG(\eta)]s^*.$$  \hspace{1cm} (14)

If $F(x)$ has an increasing generalized failure rate (IGFR), then we can further obtain that the unique optimal satisfies (see Lariviere and Porteus, 2001)

$$\frac{\partial \Pi_s(Q^*)}{\partial w} = F^{-1}\left(\frac{w + c_m}{p}\right) - \frac{w - cG(\eta)}{pf(s^*)} = 0.$$ \hspace{1cm} (15)

Compared Eq. (10) with proposition 1, we find that wholesale-price-only contract cannot coordinate the supply chain. The decentralized supply chain’s expected profit may be lower than that of the integrated supply chain. This phenomenon is well known as double marginalization (Spengler, 1950; Cachon, 2003). In order to encourage the manufacturer to order a $(s, S)$ inventory quantity, the supplier must offer a proper contract to the manufacturer so that the supply chain coordination is achieved. Actually, the singly traditional contact (such as, revenue sharing contract, buy-back contract, etc) cannot coordinate the supply chain because Eq. (2) has three decision variables. Therefore, we will adopt a combined contract to achieve coordination in next section.

**5. SUPPLY CHAIN COORDINATION**

The buy-back contract is one of the most commonly used contracts. In this section, we propose a new shortage penalty contract, and then use a contract combining buy-back and shortage penalty (BBSP) to achieve supply chain coordination. Under the $(s, S)$ inventory ordering policy, the manufacture not only face the upstream risk of shortage problem by the random yield, but also meet the downstream risk of remaining problem by the stochastic demand. Under the shortage penalty contract, the supplier will be punished for under delivery. The manufacturer sets up a delivery target $D_0(D_0 > S_0)$ and if the supplier’s delivery quantity is below the target, then he/she will pay the manufacturer a penalty for each unit of below $D_0$. Under a returns policy, the supplier pays the manufacturer back for unsold products to encourage manufacturer to order a larger quantity. Let the return credit for each unit of unsold products be $t$ where $t \in (0, w)$. Therefore, the shortage penalty could reduce the upstream risk of random yield and returns policy can decrease the downstream risk of random demand.

**5.1 The manufacture’s problem**

Under the BBSP contract, the expected profit for the manufacturer is given by

$$\Pi_m(s, S, Q) = E[p \min(D, X) - (w + c_m)D + t(D - X)^+ + \alpha(D_0 - D)^+]$$

The first term in $\Pi ms(s, S, Q)$ is the total revenue from sales, the second term is the replenishment costs for the raw material and the costs incurred by producing the end products, and the last two terms are the expected revenue from return left over in inventory and penalty the supplier has to pay for the shortage, respectively.

From the above equation, $\Pi ms(s, S, Q)$ can be rewritten as

$$\Pi_m(s, S, Q) = \int_{s/Q}^{s/q} \{ (p-t)E[\min(S, X)] - (w+c_m-t+\alpha)S \} g(y)dy$$

$$+ \int_{s/q}^{q} \{ (p-t)E[\min(yQ, X)] - (w+c_m-t+\alpha)yQ \} g(y)dy$$

$$+ \alpha D_0 G\left(\frac{D_0}{Q}\right) + \alpha \int_{D_0/Q}^{\theta} y g(y)dy.$$ \hspace{1cm} (16)
Under the BBSP contract, in order to induce the manufacture to order the \((s, S)\) inventory ordering amount as the integrated supply chain does, i.e., \((s, S) = (s^0, S^0)\), using Eqs. (16) and (2), the contract parameters need to set such that for \(\lambda > 0\),

\[
\frac{p-t}{p} = \frac{w + c_m - t + \alpha}{c + c_m} = \lambda.
\]  

(17)

Substituting Eq. (17) into Eq. (16), the expected profit for the manufacture becomes

\[
\Pi_m(s, S, Q) = \lambda \Pi_m(s, S, Q) - \lambda (cQ\mu_t - c_s Q) + \alpha D_o G \left( \frac{D_o}{Q} \right) + \alpha Q \int_{D_o/Q}^{b} y g(y) dy.
\]  

(18)

It follows immediately that \((s, S) = (s^0, S^0)\) is optimal for the manufacture.

5.2 The raw-material supplier’s problem

Similarly, under the BBSP contract, the raw-material supplier’s expected profit can be written as

\[
\Pi_s(s, S, Q) = E \left[ wD - c_s Q - c(S - cQ) I(s \geq cQ) - t(D - X)^+ - \alpha (D_o - D)^+ \right] 
\]

\[
= \int_{s}^{s_0} \{ (w - t - c + \alpha) S - t E \left[ \min(S, X) \right] \} g(y) dy 
\]

\[
+ \int_{s}^{b} \{ (w - t + \alpha - c) y Q - t E \left[ \min(yQ, X) \right] \} g(y) dy 
\]

\[
- \alpha D_o G \left( \frac{D_o}{Q} \right) - \alpha Q \int_{D_o/Q}^{b} y g(y) dy - c_s Q + c\mu_t Q.
\]  

(19)

The first- and second-order conditions of in Eq. (19) are given by

\[
\frac{\partial \Pi_s(s^0, S^0, Q)}{\partial Q} = \int_{s}^{b} \left[ t y F(yQ) - (t - w + c - \alpha) y \right] g(y) dy - \alpha \int_{D_o/Q}^{b} y g(y) dy - c_s + c\mu_t.
\]  

(20)

\[
\frac{\partial^2 \Pi_s(s^0, S^0, Q)}{\partial Q^2} = -t f(yQ) g(y) dy - \alpha \frac{D_o^3}{Q^3} g \left( \frac{D_o}{Q} \right) < 0.
\]  

(21)

Eq. (21) shows that \(\prod ms(s, S, Q)\) is concave in \(Q\). To induce the raw-material supplier to input the production quantity as the integrated supply chain does, i.e., \(Q = Q^0\), Comparing first-order condition Eq. (17) with first-order condition Eq. (5), there exists a unique \(Q = Q^0\) that maximizes \(\prod ms(s, S, Q)\) if and only if \(D_o\) is uniquely solved from the following Eq. (22).

\[
\alpha \int_{D_o/Q}^{b} y g(y) dy - c_s + c\mu_t
\]  

(22)

The above equation shows that the raw-material supplier’s target \(D_o\) will be a fixed multiplier of his/her production quantity if the supply chain coordination is achieved.

When the supply chain coordination is achieved, substituting Eqs. (22) into Eq. (18), then the expected profits for the manufacturer is
\[ \Pi_m(s^0, S^0, Q^0) = \lambda \Pi_{ms}(s^0, S^0, Q^0) + \alpha D_0 G \left( \frac{D_o}{Q^0} \right). \]  

(23)

Correspondingly, the supplier’s profit is

\[ \Pi_s(s^0, S^0, Q^0) = (1 - \lambda) \Pi_{ms}(s^0, S^0, Q^0) - \alpha D_0 G \left( \frac{D_o}{Q^0} \right). \]  

(24)

Hence, we straightforward obtain the following proposition.

Proposition 4. The BBSP contract can coordinate the supply chain if the contract parameters satisfy conditions (17) and (22), and an arbitrary allocation of the optimal supply chain profit within a certain range between the manufacturer and the supplier can be achieved by varying \( t \) and \( \alpha \).

From Proposition 4, we have

\[ \frac{dt}{d\alpha} = \frac{p}{p - c - c_m} > 0. \]  

(25)

Therefore, when the shortage penalty of the supplier increases, the return credit also increases. Indeed, this result is intuitive, since the shortage penalty increasing leads to the supplier’s revenue decreasing, the supplier can increase the return credit to decrease the manufacture’s profit proportion of the integrated supply chain and compensate the loss of increasing shortage penalty. This further illustrates that supply-chain coordination is actually a process of supply-chain risk re-allocation between members. When the supplier’s risk is more severe, the contract transfers some of this risk to the manufacturer, and the supply chain remains coordinated.

6. NUMERICAL EXAMPLES

To gain further insights, we give some numerical examples under uncertainties with and without coordination, and illustrate that our \((s, S)\) inventory ordering policy can significantly enhance the expected profit of the integrated supply chain than the invariable ordering policy does.

We assume the raw-material supply follows a uniform distribution and the final demand follows a normal distribution. Since the structure of our model is similar to that presented in He and Zhao (2012), as stated in Section 3, and we intended to compare some of our results with those in He and Zhao (2012), we adopted the basic parameters used in He and Zhao (2012) as follows: \( p=15 \), \( c=10 \), \( c_m=2 \). Supply has a mean of \( \mu_1 = 0.7 \) and a standard deviation of \( \sigma_1 = 0.1 \), and demand has a mean of \( \mu_2 = 1000 \) and a standard deviation of \( \sigma_2 = 500 \).

Figs. 1 and 2 summarize the effect of stochastic yield and demand uncertainty on supply chain’s profit. Figs. 3 and 4 show the percentage increase of the supply chain’s profit with the \((s, S)\) inventory ordering quantity policy as compared to supply chain’s profit with the invariable ordering policy in He and Zhao (2012), and this amount is calculated as: \% profit increase = \( \left( \frac{\Pi_{ms}}{\Pi_m} - 1 \right) \times 100\% \). Figure 5 and 6 show the percentage increase in supply chain’s profit under coordination as compared to supply-chain profit without coordination. This amount is calculated as: \% profit increase = \( \left[ \frac{\Pi_{ms}^{0}}{\Pi_{ms}^{0} + \Pi_s} - 1 \right] \times 100\% \) Figure 7 and 8 reveal the effect of contract parameter \( t \) on supply chain’s profit allocation and the relationship between two contract parameters when the supply chain achieves the coordination with \( w=10.55 \).
Figure 1. Impact of yield uncertainty

Figure 2. Impact of demand uncertainty

Figure 1 and 2 reveal that the integrated supply-chain profit suffers from a decrease under yield uncertainty and demand uncertainty. Figure 1 and 2 also show that an increase in demand and yield uncertainty damage the profits of supply-chain members, the exception being the manufacturer. We find that first increases and then decreases with the demand uncertainty and random yield does not have any effect on the manufacturer’s profit under the decentralized decision making. From Figure 3 and 4, we find that the profit increase becomes more obvious as the supply uncertainty or the demand uncertainty is more severe, and the rate of profit increase is more rapid as the demand uncertainty increases. These results show that our (s, S) inventory ordering policy can reduce the risk of uncertainty in the supply chain and then enhance the supply chain’s expected profit. From Figure 5 and 6, we see that coordination always benefits the entire supply chain and that the percentage of profit growth is the opposite in demand and yield uncertainty. Figure 7 shows that an arbitrary allocation of the optimal supply chain profit within a certain range between the manufacturer and the supplier can be achieved by varying contract parameters when the supply chain achieves the coordination. From Figure 8, we can find that the relationship of linear growth between the two contract parameters.
Figure 7. Impact of return credit price on profit

Figure 8. Impact of return credit on shortage penalty

7. CONCLUSIONS

This paper probes into the ordering decision of the manufacture, raw-material planning decision of the supplier, and the optimal contract for a supply chain with both demand and supply uncertainty. We propose a \((s, S)\) inventory ordering quantity for the manufacture, where \(S\) is such that the supplier’s delivery quantity is raised to the order-up-to level \(S\) when the supplier’s production decreases to a value equal to or smaller than the order level \(s\). Both the centralized decision making and decentralized decision making are studied and the analytical results are provided. The key research question asks what contract format and contract parameters perfectly coordinate the supply chain and achieve a win-win situation by adjusting the contract parameters. Our study provides the equilibrium decisions on inventory and production planning in the integrated supply chain and shows that the commonly used wholesale-price contracts cannot coordinate the supply chain in the decentralized setting. Our work reveals that a properly designed returns policy and shortage penalty between the manufacturer and the raw-material supplier can efficiently coordinate the entire supply chain. Further more, an arbitrary allocation of the optimal supply chain profit within a certain range between the manufacturer and the supplier can be achieved by varying contract parameters. Numerical examples are presented to investigate the behavior of the proposed model under both supply and demand uncertainty and to compare the results with those in He and Zhao (2012), and reveal that our policy can significantly enhance the expected profit of supply chain than He and Zhao’s policy does. Our study sheds new light on the management of supply chain with random supply and stochastic demand.

Future research in this area is rich. First, our model assumes that the supplier and the manufacturer are both risk neutral. Hence, as a natural extension of our work, it may be also worthwhile to study the proposed model under risk measures such as the expected utility objective, conditional value-at-risk and mean-variance criterion. The other possibility is to investigate more general supply chains such as multi-period models and multiple manufacture and multiple supplier models deserve further study. Finally, we can study and evaluate other contract formats, such as combining revenue sharing and shortage penalty contract or buy-back with other new ordering policy.

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APPENDIX

Proof of Proposition 1. To maximize the expected profit \(\Pi_{ms}(s, S, Q)\) in Eq. (2), taking the first-order derivatives of \(\Pi_{ms}(s, S, Q)\) with respect to \(s\) and \(S\), respectively, we obtain

\[
\frac{\partial \Pi_{ms}(s, S, Q)}{\partial s} = \frac{g(s/Q)}{Q}\left\{pE[\min(S, X)] - pE[\min(s, X)] - c_s(S - s) - c_m(S - s)\right\},
\]  

(A.1)
\[
\frac{\partial \Pi_m(s, S, Q)}{\partial S} = \int_a^{\sigma_Q} \left[ p \Phi(S) - c - c_m \right] g(y) dy. \tag{A.2}
\]

Let \( S^0 = F^{-1}[(c+c_m)/p] \), then it is obvious that \( S^0 \) satisfies the first order condition in Eq.(A.2). Further more, we have

\[
\frac{\partial^2 \Pi_m(s, S, Q)}{\partial S^2} = -p \int_a^{\sigma_Q} f(S) g(y) dy \leq 0. \tag{A.3}
\]

Hence, for any given \( s>0 \) and \( Q>0 \), there exists a unique \( S = S^0 \) which maximizes \( \Pi_{ms}(s, S, Q) \).

Let \( \frac{\partial \Pi_m(s, S^0, Q)}{\partial s} \bigg|_{s=s^0} = 0 \), then we get Eq.(4).

By taking the second-order derivatives of \( \Pi_{ms}(s, S^0, Q) \) with respect to \( s \) and \( F(s) \geq F(S) \), we have

\[
\frac{\partial^2 \Pi_m(s, S^0, Q)}{\partial s^2} \bigg|_{s=s^0} = \frac{g(s/Q)}{Q} \left[ c + c_m - p F(s) \right] \leq 0. \tag{A.4}
\]

Therefore, for any given \( Q>0 \), the optimal \((s, S)\) inventory ordering policy \((s^0, S^0)\) maximizes the expected supply chain profit.

Next, we discuss the supplier’s optimal production quantity \( Q^0 \) that maximizes the expected supply chain profit \( \Pi_{ms}(s^0, S^0, Q) \). We consider taking the first-order and second-order derivatives of \( \Pi_{ms}(s^0, S^0, Q) \) with respect to \( Q \) as follows.

\[
\frac{\partial \Pi_m(s^0, S^0, Q)}{\partial Q} = \int_{y_0}^b \left[ py \Phi(yQ^0) - (c + c_m)y \right] g(y) dy - c_s + c \mu. \tag{A.5}
\]

\[
\frac{\partial^2 \Pi_m(s^0, S^0, Q)}{\partial Q^2} = -p \int_{y_0}^b y^2 f(y) g(y) dy \leq 0. \tag{A.6}
\]

Since it is obvious that \( \frac{\partial \Pi_m(s^0, S^0, Q)}{\partial Q} \bigg|_{Q=Q^0} = c \mu - c > 0 \) and \( \lim_{Q \to \infty} \frac{\partial \Pi_m(s^0, S^0, Q)}{\partial Q} = -c_s < 0 \), there exists a unique solution \( Q = Q^0 \) such that \( \frac{\partial \Pi_m(s^0, S^0, Q)}{\partial Q} \bigg|_{Q=Q^0} = 0 \), which implies that \( Q^0 \) satisfies Eq. (5).

Proof of Proposition 2. Let \( \prod(T) = E[p_{\min}(T, X)] - (c-c_m)T \), then it is easy to verify that \( \prod(T) \) is increasing on \((0, S^0)\) and decreasing on \([S^0, \infty)\). In addition, it is easy to verify that \( s^0 < S^0 < S^1 \) and \( s^0/Q^0 < S^0/Q^0 < S^0/Q^0 \). Therefore, we obtain

\[
\Pi_m^0(s^0, S^0, Q^0) - \Pi_m^0(S^0, Q^0) = \int_a^{\sigma_Q} \left[ \pi(S^0) - \pi(S^0) \right] g(y) dy + \int_{y_0}^b \left[ \pi(yQ^0) - \pi(S^0) \right] g(y) dy + \int_{y_0}^b \left[ \pi(yQ^0) - \pi(S^0) + c(yQ^0 - S^0) \right] g(y) dy.
\]
The three terms in \( \prod_{m}^{0}(s^{0},S^{0},Q^{0}) - \prod_{m}^{0}(S^{0},Q^{0}) \) are all nonnegative, and at least one term among them is positive. Hence, \( \prod_{m}^{0}(s^{0},S^{0},Q^{0}) \geq \prod_{m}^{0}(s^{0},S^{0},Q^{0}) > \prod_{m}^{0}(S^{0},Q^{0}) \).

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