Research on the Application of Partial Differential Equation in Remote Sensing Image Denoising and Classification

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Abstract

In the process of acquisition and transmission of remote sensing images, a variety of internal and external interferences make the image contains noise. That leads to the decreasing of image quality. It is difficult to identify and analyze the remote sensing image. Therefore, it is necessary to reduce the noise of the remote sensing image before use. In recent years, image processing based on partial differential equation (PDE) has been widely used in the field of image processing. The application of PDE in the field of remote sensing image processing can solve some difficult problems that exist for a long time. This paper mainly research on the denoising and the classification of remote sensing image based on PDE method. Application of image denoising is studied with many experiments, including P-M anisotropic diffusion model proposed by Persona and Malik, the basic TV restoration model proposed by L.Rudin and S.Osher in 1992 and direction diffusion model proposed in 1974. Then, we have researched on the application of direction diffusion method based on PDE in remote sensing image classification. It is proved by experiments that the application of PDE in remote sensing image processing has a good effect. It has practical significance.

Keywords: Remote Sensing Image, PDE, Image Denoising, Image Classification.

1. INTRODUCTION

With the improvement in space, time and spectral resolution of remote sensing images, high resolution image processing and the use of remote sensing images has been applied widely now days. Besides the spectral information of low resolution multispectral images, it also has abundant spatial information and more obvious geometry and texture information. Because of the increasing of information, it is easy to recognize the attributes of objects. The extensive application of remote sensing technology promotes the research of remote sensing image processing algorithms, which makes the algorithm put forward higher requirements and challenges in the processing of remote sensing image with high efficiency and accuracy. Remote sensing image classification is one of the important fields in the research of remote sensing image processing algorithms, and it is one of the core functions of remote sensing image processing system, it is also the basic component of the application research, such as professional information extraction, thematic map making, dynamic change prediction and the establishment of remote sensing database. Classification is one of the important means for human to understand and study the use of surface information, and the accuracy of classification affects the application level and practical value of remote sensing technology (Liu et al., 2012; Zhang et al., 2011). Although new theories and algorithms in the field of remote sensing image classification emerge one after another, but the goal of improving the classification accuracy remains unchanged. It is a multi-classification problem which transforms the image data from the 2D gray space to the target pattern space. The traditional classification method based on statistics, which is mainly based on pattern recognition method, and that needs to do a lot of statistical analysis of the image and find the inherent regularity. After continuous development and improvement, the method is more mature and widely applied.

At present, the classification algorithms based on pixel statistics can be divided into supervised classification and unsupervised classification (Duarte et al., 2008; Liu et al., 2011; Tschumperl et al., 2007). Unsupervised classification algorithm does not need priori knowledge, only by analyzing the characteristics of the image itself. Compared with the corresponding algorithm of supervised classification, there is a lack of sample analysis. Supervised classification requires priori knowledge to train all kinds of samples, using the pixel characteristics of the trained and recognized samples to identify the unknown pixels, this is the common method. The main supervised classification algorithms are: minimum distance method, distance method, adjacent method, parallelepiped method, maximum likelihood method, decision tree and Markov random field model. These
algorithms are based on strict mathematical basis, but there are some shortcomings and limitations: there are problems such as large computation and complex operation in above algorithms. If the assumed probabilistic model is biased from the actual situation, the classification error occurs. It is difficult to identify the remote sensing images with complex background, target occlusion and the presence of clouds or other overcast. With the further development of remote sensing technology, some problems of statistical pattern recognition methods appear. Because of the complexity of the spectral features, the limitations of the traditional classification methods are more prominent. Since the development of pattern recognition technology in 90s, especially the breakthrough of artificial intelligence, new theories and methods have emerged and been introduced into the classification of remote sensing images. Classification methods such as neural network classification, support vector machine (SVM) classification, wavelet analysis classification, fuzzy classification, object oriented classification, knowledge based decision rule classification, multi-source data fusion classification, expert system classification appear, which improves the classification effect and classification accuracy. The classification of different remote sensing images should be used flexibly. Some classification methods may be more suitable for the classification of certain images. To explore a more appropriate and effective method is the goal of current research. Artificial neural network classification is a branch of artificial intelligence, the non-parametric classifier can transform the original feature space into a new data space. According to the spectral characteristics of remote sensing images, the pattern recognition technology is widely used in the field of remote sensing technology. Through the identification and classification of the remote sensing image, we can accurately identify all kinds of interesting objects and convert them to the desired information (Meng et al., 2015; Nazari et al., 2015; Krishnasamy et al., 2010). That can effectively improve the timeliness and accuracy of information, and achieve the purpose of extracting the needed information. However, there is still a gap between the development level of remote sensing image classification and the practical application. But with the extensive use of some powerful remote sensing professional processing software, we can quickly get the surface information and reduce the workload greatly. Artificial neural network (ANN) is a complex network composed of a large number of simple processing units, it is a kind of data analysis system which is based on the simulation of human brain nerve system. It can simulate parts of the image thinking ability, reflects the basic features of the human brain function. It is an abstraction, simplification and simulation of the human brain. It has the characteristics of parallel processing, distributed storage, self-organization, self-learning and self-adaptation. Artificial neural network is applied to image classification by Li Shihua and Zheng Guoming, which can simulate large-scale adaptive nonlinear complex system, it has the characteristics of self-organization, high dimension and redundancy of general nonlinear dynamical systems, and it can be used to solve complex and highly nonlinear classification and pattern recognition problems (Zehtabian et al., 2014; Yahya et al., 2014). The method was applied to remote sensing image classification at the end of 90s. The main methods include neural network, radial basis function neural network, fuzzy neural network, three-dimensional network, wavelet neural network, self-organizing feature classifier and other classifiers (Zehtabian et al., 2015; Wang et al., 2008; Hara et al., 2007). It can be used to identify and extract the attribute information of remote sensing image accurately and quickly, which has been widely used in remote sensing image classification. The algorithm of remote sensing image classification based on radial basis function neural network and mean value method is proposed by Wang Ling. The self-organizing network is applied in the land cover classification by Zhang Danfeng. Compared with the maximum likelihood classification applied in the high dimensional remote sensing, the accuracy of this method is much higher. However, the accuracy is obtained at the expense of increasing computational complexity. And much more iterations are needed, which makes the training cycle much longer (Ma et al., 2013; Cui et al., 2016; Jiang et al., 2012). It is necessary to do further research on establishment of the network structure, the number of hidden nodes, and the choice of parameters. We also need to improve the algorithm to adapt to the processing of massive remote sensing data.

2. CORRELATION THEORY

In the past few years, the partial differential equation has become one of the most important research fields in mathematical analysis. In image processing field, except the wavelet transform one of the important results is the development of image processing technology based on partial differential equations. Since 80s, partial differential equations have been widely used in many fields such as material science, physics mechanics, finance, biology and so on. Based on the image processing method, the variational method, functional analysis, differential geometry and projective geometry are combined to establish a set of perfect theory system (Singh et al., 2010; Wedemeyer et al., 1964). The thought of partial differential equations applied to the image processing can be traced back to the work of scale space introduced by D.C. rdbor et al. However, the substantial leap due to Koenderink and Witkin. With the concept of scale space has been introduced, image is expressed in multiple scales, and this constitutes a preliminary theoretical basis for using PDE in image processing field.

2.1 Research on PDE based on Wavelet Transform
Wavelet analysis also known as multiresolution analysis, enjoys the title of "mathematical microscope". Wavelet transform is purely mathematical and should be. It is a shining example that combining with mathematics perfect. It has important significance in the history of mathematics, numerical analysis, functional analysis, harmonic theory, approximation theory and time-frequency analysis theory and so on. From a purely mathematical point of view, the wavelet transform is a harmonic analysis including the function space, the generalized function, analytical and abstract harmonic analysis, which has been used for half a century since the application of science.

It is a major breakthrough in the application of computer, signal processing, nonlinear science and engineering technology in recent years. With the development of the research, the application of wavelet transform has become a very active research field in the world. Image denoising is an image preprocessing, the purpose is to improve the signal to noise ratio and improve image quality. At present there have been many methods of image denoising method, such as adaptive smoothing filter, wavelet transform, random variable method, Yuan Zejian method. Though they are denoising, but the intrinsic link between the little literature such as Wu Yadong’s research. Only by recognizing the nature and advantages and disadvantages of various methods, can we provide the guiding ideology for the creation of a new composite method. Image denoising method takes the edge of the image as the boundary, and uses piecewise continuous function to approximate and suppress random noise. The edge will not cause ambiguity. The noise model is given, and the filter of the denoising model is given. The soft threshold denoising and anisotropic diffusion are widely used effective two denoising methods. Which has important theoretical and practical significance to study the internal relations. Wavelet shrinkage denoising technique and nonlinear diffusion filtering are two important denoising methods. This chapter focuses on preliminary analysis between wavelet shrinkage denoising and nonlinear diffusion. Nonlinear diffusion filtering of wavelet shrinkage is given under the framework of interpretation. Then the advantages of these methods are given. In the next chapter, we propose a hybrid denoising algorithm.

2.2 Two denoising model of PDE

In recent years, a lot of people use the P-M model and the four order PDE for remote sensing image denoising. The P-M equation has a good ability to maintain the edge. While the four order PDE is very good in suppressing the block effect, which generated by denoising process. However, it is not enough to remove the mixed noise of remote sensing images. Therefore, an improved hybrid model of these two types is proposed, which is a pure anisotropic diffusion model with a new four order PDE.

2.2.1 Pure anisotropic diffusion model

Perona and Malik proposed the P-M equation in 1990, which can preserve the edge information while the image denoising. P-M model is as follows:

\[
\begin{align*}
\frac{\partial u(x, y, t)}{\partial t} &= \text{div}(g(| \nabla u |) \nabla u) \\
u(x, y, 0) &= u_0(x, y)
\end{align*}
\]  

In the formula, \( g(| \nabla u |) \) is the conduction coefficient. Perona and Malik define it as:

\[ g(| \nabla u |) = \exp\left(-\left(\frac{u}{K}\right)^2\right) \]  

But the P-M model is easy to cause “block effect”, and with the number of iterations increasing, the edge kept effects is worse. The treatment effect of salt and pepper noise is not ideal. Aim at these problems, Alvarez Proposed a pure anisotropic diffusion equation based on the mean curvature of the image in 1992. As follows:

\[
\begin{align*}
\frac{\partial u(x, y, t)}{\partial t} &= g(| \nabla * u |) | \nabla u | \text{div}\left( \frac{\nabla u}{| \nabla u |} \right) \\
u(x, y, 0) &= u_0(x, y)
\end{align*}
\]
In the formula, \( G_o = \frac{1}{4\pi\sigma^2} e^{-\frac{x^2+y^2}{4\sigma^2}} \) is kernel function of Gauss filter. \( \sigma \) is filter scale.

Discretized the diffusion equation by using the finite difference. Time step is \( \Delta t \). Due to the diffusion of the model only in a tangential direction parallel to the edge of the thousand, so it have edge protect ability and strong ability to salt and pepper noise.

2.2.2 Four order PDE model for image denoising

Yu-Li and M.Kaveh proposed an forget order model of the PDE image processing. This type of film not only can avoid the noise at the same time can avoid the two order PDE "block effect", but also can protect the edge well.

Base change problem:

\[
E(u) = \int f(|\nabla^2 u|) \, d\Omega 
\]

(4)

In 2000 Yu-Li and M. Kaveh proposed the following equation:

\[
\frac{\partial u}{\partial t} = -\nabla^2 [g(|\nabla^2 f(.)|)\nabla^2 u] 
\]

(5)

New four order PDE not only can remove the noise and preserve the boundary of Gauss. It can also remove a part of the salt and pepper noise. From the previous discussion, in this model, the impulse noise is eliminated completely and the anisotropic diffusion is preserved. But when the remote sensing image contains a large noise, it will appear the " " after several iterations. The new four order partial differential equation can protect the smooth area and eliminate the "block effect" excellent. But the ability to protect edge is weak. According to the characteristics of these two kinds of models, a remote sensing image denoising algorithm based on hybrid model is proposed.

2.2.3 Hybrid PDE model for image denoising

In the remote sensing image denoising, the image is divided into the flat region, the gradient region and the mutation region. Flat region mainly contain image signal and gauss noise, may also contain less salt and pepper noise. The gradient region is the superposition of two kinds of noises. The mutation region contains the image edge and texture features, most of the salt and pepper noise.

The anisotropic diffusion model is used in the mutation region, and new four order PDE is used in the flat region, and the gradient region is weighted by the two methods. The algorithm overcomes the traditional anisotropic diffusion model over diffusion in smooth region which cause “block effect”, and the new four order of PDE removing noise and excessive loss of edge information. This method is effective to remove the remote sensing image with Gauss and pepper mixed noise, and keep the image edge and texture details well.

Image denoising algorithm specific steps:

(1) Using the pure anisotropic diffusion denoising and the new order of four PDE denoising on the image, get two images H and I;

(2) The region segmentation of the noisy image is obtained: the flat region, the gradient region and the mutation region (the gray value of the flat region is 0, the gradient region is 128, and the mutation area is 255);

(3) Get the denoised image. In the mutation region adopt the pure anisotropic diffusion, and the new four order PDE is used in the flat region.

2.3 The remote sensing image classification of PDE
With the development of remote sensing image classification technology, there are many kinds of classification methods. They can be divide into Pattern recognition method for supervised classification and unsupervised classification of each pixel in a remote sensing image according to whether there is a sample learning or whether a priori knowledge is used. The main difference is that the top-down knowledge driven method is used in the supervised classification, that is to say, the first is to learn and then to classify the contrary. As known as the supervised classification method or training-learning classification. Using pixel characteristics have been training in samples to identify the unknown pixels. Supervised classification has a good accuracy, but it requires sampling, so it have a large workload and require selecting representative samples. That means spend a long time.

2.3.1 Unsupervised classification

Unsupervised classification, also known as edge learning or edge classification. The similarity of unsupervised classification is the feature value, without selecting the training ground to establish discriminant function. However, the brightness values of the input pixels are obtained by statistical analysis. It gives a number of categories of multidimensional space, each of which is composed of clusters of images, and then the classification of remote sensing images.

The K-mean classification method assumes that the number of cluster centers that are used to represent the sample space. And it is restricted. Based on the principle of least square error, the basic idea is to get the best clustering result by moving the center of each benchmark. Finally get the best clustering result. As shown in Figure 1.

![Figure 1. The k-means clustering](image)

2.3.2 Supervised classification

Supervised classification can be determined the classification category through the human selection, and avoid some unnecessary categories by using prior information. According to the purpose and the research area, the training area is selected artificially. By checking the training area to determine whether the pixels to be classified accurately. In this way, we can avoid serious errors. Therefore, it has strong subjective factors. In order to select and evaluate the training area, we have to spend more time and manpower. Due to the natural categories may exist not in any studied areas, the training area of the artificial selection may not represent the real situation in the image effectively. Because the same object is divided into different categories and different objects are divided into the same category, so the classification results can not be a good representative of the category. Some special categories cannot be defined, either because the trainers don't know how to define the category or because the number of samples is too small, at this time, the supervised classification cannot be identified. The most common supervised classification algorithms include minimum distance classification,
parallel pipeline classifier, maximum likelihood classification, neural network classifier, support vector machine and so on.

The minimum distance classifier is used to calculate the standard deviation of the mean vector and vector of the training samples. According to the distance between the sample and the center of the class. The distance from which center is smallest, the sample belongs to which category. In this method, the distance is the only criterion. In the feature space, pixels is classified to each class center by distance has been calculated. The disadvantage of the minimum distance classifier is that the distribution of the point distribution of natural classes is different in different directions, and it is not necessarily circular or spherical. Because of the variance of the variance in each direction, we can not simply use the pixel distance as the basis for classification. When a large number of non-uniform distribution exist, the classification performance will become very poor. As shown in Figure 2.

![Figure 2. The minimum distance classifier](image)

3. RESULTS AND DISCUSSION

In this paper, the simulation experiment is carried out in Matlab7.9 environment. In order to verify the effectiveness of remove CCD noise by hybrid PDE method based on region segmentation. We select the remote sensing image of Athens Olympic center. Intercept a part of 512 x 512. In the part of the image by adding a Gauss noise($a=25$) and pepper noise with density of 0.04%, the number of iterations is 50, the time step is 0.2, using the method proposed in this paper experiments on the part of remote sensing image plus noise. The results are shown in figure 3.

![Figure 3. The verify of the effectiveness of algorithm](image)

As can be seen from Figure 3, after the denoising method. Image clarity significantly enhanced. The signal noise ratio of the original image is 2.5266dB. The signal to noise ratio after denoising is 7.8777dB. The experimental results show that the proposed method is effective for removing CCD noise.

In order to verify the effectiveness of the proposed method for remote sensing image denoising. In this paper, the use of LANDSAT7 remote sensing images to verify. Intercept of its 512 X 512 image denoising. After a number of experiments, the simulation time step $DT = 0.2$, the number of iterations = 30 conduction coefficient $K=15$ in the best denoising effect. The use of pure anisotropic diffusion Lam, four order PDE model and the proposed hybrid PDE model based on region segmentation of remote sensing image denoising respectively. The experimental results are shown in figure 4.
Can be seen from Figure 4, the anisotropic diffusion model and the four order PDE model combining while filtering the image noise, but the local smoothing phenomenon, while this algorithm remains on the edge of the superior to the other algorithms, the image noise was filtered, and the clear edge details. The above two algorithms are respectively calculated SNR, definition, operation time of the image, as shown in Table 1.

<table>
<thead>
<tr>
<th>Method</th>
<th>SNR</th>
<th>resolution</th>
<th>running time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hybrid model</td>
<td>22.5852</td>
<td>9.7425</td>
<td>23.19996</td>
</tr>
<tr>
<td>Paper algorithm</td>
<td>25.5649</td>
<td>10.8265</td>
<td>11.41016</td>
</tr>
</tbody>
</table>

It can be seen from table 1, using remote sensing image mixed PDE segmentation denoising algorithm compared with other algorithms based on the SNR and the clear degree have great lift off.

Neural network approach for supervised classification. Figure 5 shows the classification results, the classification results for the area roughly include buildings, woodland, grassland and bare land, rivers and so on, it can also be more classification to determine the objects more. Due to the limited space in this paper, the image is cut, and some representative regions are reserved.

With the increase of the number of iterations, a large number of details of the image began to decrease, and a small area of the map began to disappear. The image edges are preserved, so this paper provides a more
effective solution. It can be said that the diffusion method can replace the classification work to a certain extent. Back to the results with the removal of noise and clustering, the direction of diffusion iteration 50 times can achieve the same effect. When the results of the 100 iteration, you can see the difference is not obvious.

4. CONCLUSION

In fact, the performance of any sonar system is affected by the background noise. The noise can be divided into three types. They are environmental noise, reverberation noise and system's noise. Before the sonar image recognition and classification, it is necessary to filter the noise to remove the interference information and preserve the target feature information. The traditional image denoising methods are mainly based on two types of spatial and frequency domain. The spatial domain method mainly used all kinds of smoothing function convolution processing denoising method in frequency domain is that the image noise component in the transform domain within the scope of a suppressed while the other component is not affected by the image, thus changing the frequency distribution of the output image denoising. Wavelet transform is an effective method for the analysis of spatial and temporal scales, which has been widely concerned in recent years. It provides a multi-resolution method for image processing. Partial differential equation method has been widely used in recent years in the field of image processing, the basic principle of denoising is using continuous piecewise continuous function approximation of the image. Denoising stability is better, but as it relates to the calculation of higher order nonlinear equations, the method of computation and real-time processing is not suitable for the image, this paper studies the lifting wavelet transform based on directional diffusion algorithm can increase the speed and reduce the number of iterations. At present, the image denoising based on partial differential equation (PDE) is mainly concentrated in the radar. This study fills the blank of the calculation of time and it is necessary to study further the algorithm of image denoising is not the need to further carry out the work, in order to get better and more efficient method. According to the characteristics of the sonar image, the paper studies the noise formation process and so on. Performance spectrum in remote sensing image value is mixed, affected by various factors, the same spectrum "and" synonyms spectrum "phenomenon is more obvious, independent component analysis of the phenomenon to a certain extent eased by nonlinear decorrelation and independent image of each band signal to solve the mixed spectral value the problem, of course only spectral classification accuracy has its shortcomings, but this is also the traditional classification method has been used in the method, of course, has its own advantages and more automatic, object oriented classification method can also exist some problems, such as the existence of multi-scale segmentation over segmentation and under segmentation problem, need people to participate more in later classification. In this paper, we use the traditional method to diffuse the pixels in the same region, and provide a good condition for the next step. Through the analysis of high resolution remote sensing image classification and traditional classification results, you can see part of the shape of objects is not rules, the classification results are scattered, mainly because of the noise of high resolution remote sensing image itself, to classify the preprocessing filter changes or denoising, although be improved to some extent, but not fundamentally resolved. This method from another way of using the method of directional diffusion, can effectively remove the noise problem and protect the classification object edge does not suffer too much influence, improve the classification accuracy of high resolution remote sensing image.

In this paper the process of classification only the use of spectral information of the image. Information for other high resolution remote sensing images are shown, such as shape, texture and context information are not use. This method does not consider the use of high-resolution remote sensing images show some of the features, and may weaken the characteristics, such as texture information. In fact, due to the high resolution remote sensing images show a lot of information, and we do not make good use of the information in the actual classification. In order to make full use of the high resolution remote sensing images to show the information to make the classification results closer to the real objects, this paper is the further research work. From the view of classification accuracy, the method of orientation diffusion based on work is more favorable to improve the classification accuracy of the two kinds of images. But will be affected by the gradient threshold size, time step size and the number of iterations, the number of iterations of gradient threshold from experimental study of small value, small time step and fewer, more close to the original image, but the actual comprehensive mapping, and perhaps not what we need, how to select the appropriate size that is also a need for a study of the problem. Improvement of the diffusion algorithm has been mainly on the stop function were improved, at present the only improvement, this paper studies are limited to the ways of the improvement of the algorithm, of course, how to use the improved better.

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