Multifractal Detrended Cross-correlation Analysis of Traffic Time Series

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Abstract
In order to reflect the trend and volatility of traffic flow state, from the global and local cross-correlation, micro time headway and velocity taken as the research object, the method of Multifractal Detrended Cross-correlation Analysis (MF-DCCA) is used to deeply analyze essential reasons and inherent dynamics mechanism of various traffic phenomena. Considering the Hurst exponent changing with different scales, MF-DCCA can not only analyze the local cross-correlation of different time intervals, but also effectively distinguish the influence of local cross-correlation and multifractal characteristics between multiscale intervals of different density. And further through differences between the global generalized auto-correlation Hurst exponent and global generalized cross-correlation Hurst exponent, the reinforcing or weakening change has been analyzed between linkage behavior and auto-correlation behavior.

Key words: Traffic time series, Multifractal detrended cross-correlation analysis, Time headway, Cellular automaton

1. INTRODUCTION
Transportation system is a complex nonlinear system. Various nonlinear dynamic phenomena of traffic flow time series can be observed, such as fractal and chaos (Wang and Shang, 2014; Li and Jiang, 2016), etc., and its characteristics and formation mechanism have always been the hot spot of traffic field (Yin and Shang, 2015). Fractal theory is used to describe the similarity of the time series in different scales, which has been successfully applied in physics, medicine, biology, geology, computer science, and many other fields.
Based on fractal theory, Detrended Fluctuation Analysis (DFA) (Peng and Buldyrev, 1994) is widely used in long-range power-law correlation analysis of non-stationary signal. For the auto-correlation of a single time series, DFA method and Multifractal Detrended Fluctuation Analysis (MF-DFA) (Kantelhardt and Zschiegner, 2012) are usually used for fractal and multifractal analysis. For the cross-correlation between two time series, Detrended Cross-Correlation Analysis (DCCA) (Podobnik and Stanley, 2008) and Multifractal Detrended Cross-correlation Analysis (MF-DCCA) (Zhou, 2008) are usually used for fractal and multifractal analysis. Based on the fractal research, the complexity and the correlation of traffic flow, a large number of studies have shown that fractal research in traffic flow can better understand the characteristics of the traffic system. MF-DFA was used to analyze the complexity of the traffic flow time series by Wu (Wu and Xu, 2011) through scaling exponent, to reveal the long-range and short-range correlation of traffic flow time series. Flow, speed and time headway are the basic statistics of traffic state. Flow is based on the measurement period, while speed and time headway are based on point-in-time. The time headway is an important evaluation index for driving safety, representing the time difference of the front vehicle and the following vehicle through the same site (Vogel, 2003).

Here, micro time headway and velocity are taken as the research object, in order to analyze the complexity and cross-correlation of traffic flow time series, in theory based on multifractal theory and in method, there are both qualitative and quantitative methods. Spatiotemporal diagrams vividly describe the evolution of traffic flow characteristics of time and space, belonging to the qualitative methods. MF-DCCA belongs to quantitative methods. Based on MF-DCCA, local cross-correlation between the time headway and velocity series in different time scales will be quantitatively analyzed and can effectively distinguish local cross-correlation and multifractal characteristics in different time scales under different density. Further, the relationship of the global generalized auto-correlation and cross-correlation Hurst exponent will be also studied.
2. METHOD AND SIMULATION MODEL

2.1. Multifractal Detrended Cross-correlation Analysis

The cross-correlation between two time series has become a research hotspot. Cross-correlation analysis can describe the interdependent relationship of series in different time, mainly including auto-correlation and cross-correlation analysis. The DFA and DCCA method is generally used to analyze the auto-correlation and cross-correlation of a single time series. Based on DCCA and MF-DFA, MF-DCCA is proposed to reveal multifractal characteristics of the cross-correlations between two non-stationary signals.

The MF-DCCA procedure consists of five steps:

Consider two time series \( \{x(i)\} \) and \( \{y(i)\} \) \( (i = 1, 2, ..., N) \), where \( N \) is the length of the time series.

Step 1: Construct the profile \( X(i) \) and \( Y(i) \)

\[
x(i) = \sum_{k=1}^{N} (x_k - <x>) \quad , \quad y(i) = \sum_{k=1}^{N} (y_k - <y>)
\]

Where \( <x> = 1/N \sum_{k=1}^{N} x_k \), \( <y> = 1/N \sum_{k=1}^{N} y_k \).

Step 2: Cut the profile \( X(i) \) and \( Y(i) \) into \( \lfloor N/s \rfloor \) non-overlapping segments of equal length \( s \).

Consider that the series length \( N \) need not be a multiple of the time scale \( s \), a short part at the end of the profile will remain in most cases. In order to retain the part of the series, the same procedure is repeated starting from the other end of the series. Thus \( 2N_s \) segments are obtained together. In practice, it is reasonable to take \( 10 < s < N/5 \).

Step 3: Calculate the local trends \( \tilde{X}_v(i) \) and \( \tilde{Y}_v(i) \) for each segment \( v \) by a least-square fit of each series \( \{X(i)\} \) and \( \{Y(i)\} \). Then we calculate the difference between the original time series and the fits.

\[
f^2(v, s) = \frac{1}{s} \sum_{i=1}^{s} |X[((v-1)s+i) - \tilde{X}_v(i)]| |Y[((v-1)s+i) - \tilde{Y}_v(i)]|
\]

for \( v = 1, 2, ..., N_s \).

\[
f^2(v, s) = \frac{1}{s} \sum_{i=1}^{s} |X[N - ((v-N_s)s+i) - \tilde{X}_v(i)]| |Y[N - ((v-N_s)s+i) - \tilde{Y}_v(i)]|
\]

for \( v = N_s + 1, N_s + 2, ..., 2N_s \).

Step 4: Average all segments to obtain the \( q \)-order fluctuation function

\[
F_q(v, s) = \left\{ \begin{array}{ll}
\frac{1}{2N_s} \sum_{i=1}^{2N_s} (f^2(v, s))^{q/2} & q \neq 0 \\
\exp \left[ \frac{1}{4N_s} \sum_{i=1}^{2N_s} \ln(f^2(v, s)) \right] & q = 0
\end{array} \right.
\]

Step 5: Determine the scaling behavior of the fluctuation functions by analyzing log-log plots \( F_q(v, s) \) versus \( s \):

\[
F_q(v, s) \propto s^{\lambda_q}
\]

The scaling exponent \( \lambda_q \) is called generalized cross-correlation exponent, describing the power-law relationship between two time series. For positive \( q \), \( \lambda_q \) describes the scaling behavior of the segments with large fluctuations. On the contrary, for negative \( q \), \( \lambda_q \) describes the scaling behavior of the segments with small fluctuations.

Moreover, a constant \( \lambda_q \) indicates monofractality, while \( \lambda_q \) is dependent on \( q \), the cross-correlations between two time series are multifractal.

The cross-correlation between time series is characterized by Hurst exponent \( H_{xy} = \lambda_q(q = 2) \), which varies from \( 0 < H_{xy} < 1 \). For long-range correlation, \( 0.5 < H_{xy} < 1 \), \( H_{xy} = 0.5 \) for uncorrelation and \( 0 < H_{xy} < 0.5 \) for long-range anti-correlation behavior between two time series. \( H_{xy} = 1 \), for \( 1/f \) noise. In
particular if two time series are same, MF-DCCA is equivalent to MF-DFA.

2.2 Brake Light Cellular Automaton Model

Traffic flow theory mainly includes fundamental diagram approach and three phase theory. As the traditional theory of traffic flow, fundamental diagram approach divides traffic flow into free flow and congested flow. And the three phase theory further divides the congested flow into synchronized flow and wide moving jam, and synchronized flow is the most complex traffic state. In this paper, the brake light cellular automaton model based on the three phase theory is used to simulate the real traffic flow. The first brake light model (BL) was proposed by Knospe etc. Jiang and others on the basis of BL Model put forward the MCD Model (Modified Comfortable Driving Model) (Jiang and Wu, 2003). The essential difference is that MCD Model belongs to the three phase traffic flow theory, while BL Model belongs to the fundamental diagram approach. Tian (Tian and Jia, 2009) has improved MCD model rules of brake light, more consistent with the actual: only when the vehicle slows down, brake light is on; different deceleration is taken under different driving conditions. Rules of Tian model are as follows:

(1) Acceleration:

\[ \begin{align*}
& \text{if } (t_k \geq t_e \text{ and } h_{\text{n}}(t) = 0 \text{ and } v(0) > 0) \text{ then } \\
& v_{n+1} = \min(v_n + a_1, v_{\text{max}}) \\
& \text{else if } (h_{\text{n}}(t) = 0 \text{ or } t_k \geq t_e \text{ and } v(0) > 0) \text{ then } \\
& v_{n+1} = \min(v_n + a_2, v_{\text{max}}) \\
& \text{else if } v(0) = 0 \text{ then } \\
& v_{n+1} = \min(v_n + a_3, v_{\text{max}}) \\
& \text{else } v_{n+1} = v_n
\end{align*} \]  

(6)

(2) Slowing down:

\[ v_{n+1} = \min(q^\text{eff}, v(t+1)) \]  

(7)

(3) Set randomization probability \( p \), randomization deceleration speed \( \Delta v \):

\[ p(v_n(t), h_{\text{n}}(t), t_k, t_e, t_d) = \begin{cases} 
 p_1 & \text{if } h_{\text{n}}(t) = 1 \text{ and } t_k < t_e \\
 p_2 & \text{if } v_n(t) = 0 \text{ and } t_k \geq t_e \\
 p_3 & \text{in all other cases}
\end{cases} \]  

(8)

\[ \Delta v(v_n(t), h_{\text{n}}(t), t_k, t_e, t_d) = \begin{cases} 
 d_1 & \text{if } h_{\text{n}}(t) = 1 \text{ and } t_k < t_e \\
 d_2 & \text{if } v_n(t) = 0 \text{ and } t_k \geq t_e \\
 d_3 & \text{in all other cases}
\end{cases} \]  

(9)

(4) Randomization:

\[ \text{if } (\text{rand}() < p) \text{ then } v_{n+1} = \max(v_n(t) - \Delta v, 0) \]  

(10)

(5) Brake light state:

\[ \begin{align*}
& \text{if } (v_{n+1} < v_n(t)) \text{ then } b_n(t+1) = 1 \\
& \text{if } (v_{n+1} \geq v_n(t)) \text{ then } b_n(t+1) = 0
\end{align*} \]  

(11)

(6) \( t_d \):

\[ \begin{align*}
& \text{if } v_n(t) = 0 \text{ then } t_d = t_d + 1 \\
& \text{if } v_n(t) > 0 \text{ then } t_d = 0
\end{align*} \]  

(12)

(7) Car motion:

\[ x_n(t+1) = x_n(t) + v_n(t+1) \]  

(13)

3. SIMULATION RESULTS AND ANALYSIS

Based on the Tian model, with periodic boundary conditions and initially uniform distribution, the road is
subdivided into 10000 cells. The length of each cell is 1.5 m, while the length of each vehicle is 7.5 m, so it means that five cells are occupied by one vehicle. Maximum speed is $v_{\text{max}} = 20$ (actual speed is 108 km/h). The corresponding parameter is that $p_b = 0.94$, $p_0 = 0.5$, $p_d = 0.1$. Density is respectively 0.03, 0.06, 0.1, 0.2, 0.4, 0.6. Sampling method is that: the cellular on the left side of the road is as sample point, and the sampling step begins at the 50,000th time step, to continuously collect 30,000 samples and to generate the corresponding time headway series under each density. As a result of too many samples, Fig. 2 lists only 2000 sampling data. Spatiotemporal diagrams are shown in Fig. 1. Free flow is illustrated in Fig. 1(a). Light synchronized flow is illustrated in Fig. 1(b) and Fig. 1(c). Heavy synchronized flow is illustrated in Fig. 1(d) and Fig. 1(e). And wide moving jam is illustrated in Fig. 1(f).

3.1 Spatiotemporal Diagrams

![Spatiotemporal Diagrams](image)

Figure 1. Spatiotemporal diagrams of Tian model (road location on the horizontal axis, and vehicle travel from left to right; time step on the vertical axis)
3.2 Time Headway and velocity Series

The diagrams show the time headway and velocity series for different values of $\rho$:
- $\rho = 0.03$ (top row, left and center)
- $\rho = 0.06$ (top row, right)
- $\rho = 0.1$ (middle row, left and center)
- $\rho = 0.2$ (middle row, right)
- $\rho = 0.4$ (bottom row, left and center)

The plots illustrate the variation in time headway and velocity over time for each value of $\rho$. The y-axis represents the time headway and velocity, while the x-axis represents time.
The change of spatiotemporal diagrams of the headway and velocity is shown in Fig. 1. It is found that when free flow ($\rho = 0.03$) appears, the vehicle with the largest speed basically remains unchanged, and gaps between vehicles are basically unchanged. When synchronized flow ($\rho = 0.06, 0.1, 0.2, 0.4$) appears, the velocity has varied more dramatically than the time headway. When wide moving jam ($\rho = 0.6$) appears, at multiple sample points, the velocity and the time headway have both varied more dramatically. So we have preliminary found that the correlation is stronger between the time headway and velocity series.

3.3 Multifractality Analysis Based on MF-DCCA

For the time headway and velocity series, in order to capture the local and global trend and cross-correlation in different time intervals, it is needed to quantitatively analyze long-range or short-range cross-correlation and multifractal characteristics between two time series through the local and global Hurst exponent, based on MF-DCCA method. According to the formula (1)-(5), to analyze the time headway and velocity by MF-DCCA, it is needed to consider how to set the scale range. It is found that if a division is less than 50 data windows, fluctuation function curves will converge at the saturation scale (Wang and Shang, 2014). Therefore, we set the range of scales in this study to be $s \in [20, N/50]$, where $N$ is the length of time series. The length of the traffic time series analyzed here is about 30,000.

In order to characterize the cross-correlations, we show the log-log plots of fluctuation function $F_{xy}(q, s)$ versus time scale $s$ between the time headway and velocity series variation in Fig 3 (polynomial order $k = 2$) by MF-DCCA. The curves correspond to $q (q = -10, -5, -3, -2, 0, 2, 3, 5, 10)$ from lower points to upper points.

In free flow ($\rho = 0.03$), $q \in [-10, -2]$, in 80 seconds ($4.32 < \log(s) < 6.02$), there exists strong power law cross-correlated behavior between two series, indicating that persistent and small fluctuations exist. The fluctuation function $F_{xy}(q, s)$ fluctuates around zero when $q \in [2,10]$, for the small and large scales $s$, indicating that large fluctuations are not obvious and the cross-correlations disappear.

In synchronized flow ($\rho = 0.06, 0.1, 0.2, 0.4$) and wide moving jam, for $q \in [-10,10]$, with the increase of scale $s$, fractal and scale-less intervals exist, and the slope of scale intervals is relatively clear, indicating that there exists strong power law cross-correlated behavior between two time series.
In order to distinguish the local cross-correlation behaviors in different scale ranges, based on MF-DCCA, in three time intervals (less than 5 minutes, more than 5 minutes but less than 15 minutes, and more than 30 minutes), we compute the local cross-correlated scaling exponents in different density. Due to the different time of collecting 30,000 samples under different density, specific comparison for scale ranges and time intervals are shown in Table 1. Fig. 4 shows the variation trend of local cross-correlation generalized Hurst exponent in different time intervals under different density. For $q=2$, the local cross-correlated Hurst exponent of different time intervals is shown under different density in Table 2.

Table 1. Comparison between multiple scales and time intervals

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>time&lt;5min</th>
<th>5min~15min</th>
<th>time&gt;30min</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>$4.321 &lt; \log(s) &lt; 6.523$</td>
<td>$6.523 &lt; \log(s) &lt; 6.965$</td>
<td>$\log(s) &gt; 7.247$</td>
</tr>
<tr>
<td>0.06</td>
<td>$4.321 &lt; \log(s) &lt; 6.643$</td>
<td>$6.643 &lt; \log(s) &lt; 7.076$</td>
<td>$\log(s) &gt; 7.409$</td>
</tr>
<tr>
<td>0.1</td>
<td>$4.321 &lt; \log(s) &lt; 6.686$</td>
<td>$6.686 &lt; \log(s) &lt; 7.129$</td>
<td>$\log(s) &gt; 7.467$</td>
</tr>
<tr>
<td>0.2</td>
<td>$4.321 &lt; \log(s) &lt; 6.686$</td>
<td>$6.686 &lt; \log(s) &lt; 7.129$</td>
<td>$\log(s) &gt; 7.523$</td>
</tr>
<tr>
<td>0.4</td>
<td>$4.321 &lt; \log(s) &lt; 6.686$</td>
<td>$6.686 &lt; \log(s) &lt; 7.129$</td>
<td>$\log(s) &gt; 7.467$</td>
</tr>
<tr>
<td>0.6</td>
<td>$4.321 &lt; \log(s) &lt; 6.584$</td>
<td>$6.584 &lt; \log(s) &lt; 7.022$</td>
<td>$\log(s) &gt; 7.357$</td>
</tr>
</tbody>
</table>
Table 2. Local cross-correlation Hurst exponent in different time intervals

<table>
<thead>
<tr>
<th>ρ</th>
<th>time&lt;5min</th>
<th>5min~15min</th>
<th>time&gt;30min</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>0.489</td>
<td>0.217</td>
<td>0.256</td>
</tr>
<tr>
<td>0.06</td>
<td>0.621</td>
<td>0.633</td>
<td>0.267</td>
</tr>
<tr>
<td>0.1</td>
<td>0.686</td>
<td>0.660</td>
<td>0.596</td>
</tr>
<tr>
<td>0.2</td>
<td>0.858</td>
<td>0.675</td>
<td>0.605</td>
</tr>
<tr>
<td>0.4</td>
<td>0.785</td>
<td>0.662</td>
<td>0.632</td>
</tr>
<tr>
<td>0.6</td>
<td>1.154</td>
<td>1.02</td>
<td>0.864</td>
</tr>
</tbody>
</table>

(a) ρ = 0.03  
(b) ρ = 0.06  
(c) ρ = 0.1   
(d) ρ = 0.2
(1) In the free flow ($\rho = 0.03$), in 5 minutes, when $q > 0$, $h(q)$ decreases from 0.9 to 0.4, and for $q = 2$, local cross-correlation Hurst exponent is 0.489 indicating that the short-range cross-correlated behavior with large fluctuations is anti-persistent and multifractal. When $q < 0$, $h(q)$ decreases from 7.5 to 4.47 indicating that the short-range cross-correlated behavior with small fluctuations is strongly multifractal. In more than 5 minutes but less than 15 minutes, for $q \in [-10, 10]$, $h(q)$ decreases from 0.31 to 0.12 indicating that the short-range cross-correlated behavior is anti-persistent and weakly multifractal. In more than 30 minutes, for $q \in [-10, 10]$, $h(q)$ fluctuates around 0.26 with small fluctuations, indicating that the long-range cross-correlated behavior with large and small fluctuations is anti-persistent and weakly multifractal.

(2) As the light synchronized flow ($\rho = 0.06$) first appears, in less than 5 minutes and more than 5 minutes but less than 15 minutes, $h(q)$ varies when $q$ varies from -10 to 10, exhibiting the multifractal feature of cross-correlated behavior. In addition, for $q < 0$ and $q > 0$, the local scaling exponents at three time intervals are more than 0.5 indicating that the cross-correlated behavior between two time series with large and small fluctuations is persistent in the short-term. In more than 30 minutes, $h(q)$ decreases from 0.29 to 0.21, and local cross-correlation Hurst exponent is smaller than 0.5 indicating that the long-range cross-correlated behavior is anti-persistent and multifractal.

(3) In the light synchronized flow ($\rho = 0.1$) and heavy synchronized flow ($\rho = 0.2, 0.4$), in 5 minutes, more than 5 minutes but less than 15 minutes, and more than 30 minutes, $h(q)$ varies when $q$ varies from -10 to 10, exhibiting the multifractal feature of long-range and short-range cross-correlated behavior. In addition, for $q < 0$ and $q > 0$, the local scaling exponents in three time intervals are more than 0.5 indicating that the cross-correlated behavior of large and small fluctuations is persistent in the long-term and short-term.

(4) In the wide moving jam ($\rho = 0.6$), in 5 minutes, more than 5 minutes but less than 15 minutes, and more than 30 minutes, $h(q)$ varies when $q$ varies from -10 to 10, exhibiting the multifractal feature of long-range and short-range cross-correlated behavior. We can point out that in 5 minutes Hurst exponent is more than 0.5 indicating that the cross-correlated behavior between two series is persistent, but no power-law cross-correlation. In more than 5 minutes but less than 15 minutes, there exists $1/f$ noise. In more than 30 minutes, there exist long-range cross-correlations.

Based on the MF-DCCA, the local cross-correlation of the time headway and velocity series can be quantitatively analyzed. Long-range and short-range cross-correlation described by local cross-correlation Hurst exponent can better reflect variation characteristics of traffic flow, and can further effectively distinguish the local cross-correlation and multifractal features in different scale ranges under different density.

The higher local cross-correlation Hurst exponent is, the stronger the long-range cross-correlation of traffic flow is, so the influence of traffic flow series becomes greater. Therefore at a certain long and persistent term, behaviors of the time headway and velocity fluctuation can better be distinguished.

In order to capture the global trend and cross-correlation under different density, the whole time headway and velocity series are used to calculate the global cross-correlation Hurst exponent, to describe the global
long-range cross-correlated behavior. On one side, the higher global cross-correlation Hurst exponent is, the stronger the global cross-correlation between traffic flow series is, and the randomness is lower. The other side, in order to study the global cross-correlation and auto-correlation characteristics with large and small fluctuations between the time headway and velocity series, global generalized auto-correlated and cross-correlated Hurst exponent are further analyzed, as shown in Fig. 5.

A. Global cross-correlation Hurst exponent $H_{xy}$

1. In free flow ($\rho = 0.03$), for $q = 2$, $H_{xy}$ is 0.339, indicating that there exist long-range anti-persistent cross-correlations between two time series. If the velocity increases, the time headway reduces.

2. In light synchronized flow ($\rho = 0.06, 0.1$), $H_{xy}$ is respectively 0.545 and 0.659. It indicates that cross-correlation Hurst exponent of the light synchronized flow is less than that of the heavy synchronized flow. And the larger the global cross-correlation Hurst exponents are, the stronger the whole interaction influence between vehicles is, which indicates that there exist long-range persistent cross-correlations between two time series. If the variation of velocity increases, the variation of time headway also increases.

In the light synchronized flow, $H_{xy}$ increases with the increase of density while $H_{xy}$ decreases with the increase of density in the heavy synchronized flow.

3. In the wide moving jam ($\rho = 0.6$), global cross-correlation Hurst exponent is 1.009, indicating that there exists $1/f$ noise.

B. Global generalized auto-correlation Hurst exponent and Global generalized cross-correlation Hurst exponent

In Fig. 5, $h_{xx}(q)$ of the time headway, $h_{yy}(q)$ of the velocity, both called the global generalized auto-correlation Hurst exponent, while $\lambda_{q}$ called global generalized cross-correlation Hurst exponent between the time headway and the velocity, all exhibit the nonlinear behavior when $q$ varies from -10 to 10, indicating that the multifractal feature of global cross-correlations exist between them. We also calculate $h_{xy}(q)$ (the average of the generalized Hurst exponents of individual time series), defined as

$$h_{xy}(q) = (h_{xx}(q) + h_{yy}(q))/2$$

4. When $q > 0$, in free flow, $\lambda_{q}$ is equal to $h_{xy}(q)$, implying that the process of two time series is ruled by similar fractal dynamics when large fluctuations appear. In synchronized flow and wide moving jam, $\lambda_{q}$ is more than $h_{xy}(q)$, indicating that the global cross-correlations of two time series is stronger than the average of auto-correlation behavior for each series when large fluctuations appear. It is concluded that the linkage behavior between two series is stronger than the auto-correlation behavior of each series. When $q < 0$, in synchronized flow, with small fluctuations, there is a trend of decreasing the interaction in the linkage behavior between two series, but in the free flow and wide moving jam, the influence is not obvious.
4. CONCLUSION

For traffic flow, the trend and volatility described by long-range and short-range correlation can reflect global and local traffic flow characteristics. And it is significant and necessary to deeply understand the complicated traffic system dynamics mechanism, in order to establish efficient and intelligent urban traffic management and control mechanism, to effectively improve the traffic conditions.

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