Adaptive Robust Control for the Linear Motion of a Spherical Rolling Robot

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Abstract
In this study, an adaptive decoupled sliding mode control scheme for set-point regulation of the linear motion of a spherical rolling robot is presented. The dynamic equations of the linear motion of the spherical rolling robot is deduced by using the Lagrangian method. By applying appropriate transformations, the equations of motion of the robotic system are represented in state space form. Based on the state space model of the underactuated spherical rolling robot, the whole system is decoupled into two second-order subsystems. A pair of first-layer sliding surfaces is defined for the two separate subsystems, and the second-layer sliding surface is designed through the use of a differentiable intermediate variable. The proposed control law is developed on the basis of the nominal model of the system and the second-layer sliding surface, and an adaptation law is derived to tune the switching gain of the decoupled sliding mode controller. The proposed control law can guarantee the asymptotic stability of the closed-loop control system. Simulation results are provided to demonstrate the effectiveness and robustness of the proposed control scheme.

Key words: Spherical Rolling Robot, Linear Motion, Underactuated, Adaptive Decoupled Sliding Mode Control, Robustness.

1. INTRODUCTION
Spherical rolling robot is a novel type of mobile robot which has a spherical external shape and moves by rolling over surfaces. Usually, spherical rolling robots are propelled by internal driving mechanisms such as unbalance masses (Tomik, F. et al., 2012), rotors (Morinaga, Svinin and Yamamoto, 2014), gyros (Urakubo T. et al., 2016), counter-weight pendulums (Roozegar M. et al., 2016), and circular plates (Sadeghian and Masouleh, 2016). In recent years, spherical rolling robots have received considerable attention from the researchers worldwide owing to the nature of their mobility. Spherical rolling robots provide significant merits over other types of mobile robots on account of their good dynamic stability, high maneuverability, and low rolling resistance. Moreover, a spherical rolling robot is light-weight, compact and has a well-sealed structure. Due to the above advantages of spherical rolling robots, they are preferably used for many different applications such as patrol and surveillance, search and rescue, hazardous environment detection, underwater monitoring, and even extraterrestrial exploration.

Generally, spherical rolling robots are underactuated mechanical systems having fewer control inputs than their degrees of freedom. However, the underactuation property and highly coupled complex dynamics of spherical rolling robots make the control design of this type of robotic systems quite difficult. Linear motion is one of the basic forms of locomotion for spherical rolling robots. With respect to linear motion control of spherical rolling robots, a number of studies concerning stabilization, set-point regulation, and tracking control problems have been made in the literature. Zhao and Wang et al. (Zhao B. et al., 2011) proposed a Gaussian function based smooth trajectory control method for set-point regulation of the linear motion of a spherical rolling robot. The desired trajectory of the pendulum swing angle is derived based on the analysis of the system dynamics, and a PID controller is designed to achieve tracking control of the pendulum swing angle. Cai and Zhan et al. (Cai, Zhan and Xi, 2011) proposed a neural network control approach for stabilization and tracking control of the linear motion of a spherical rolling robot. The proposed control structure consists of a PID controller utilized to guarantee the system stability and a neural network controller employed to compensate for the actuator nonlinearities. However, these studies have not considered the effects of parametric uncertainties and external disturbances in the dynamics of the robotic system.

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Sliding mode control is one of the effective robust control approaches for nonlinear systems, since it provides the system dynamics with an invariance property against system uncertainties and external disturbances once the system dynamics are transitioned into sliding mode. Due to this distinguished feature of sliding mode control, it has been applied to linear motion control of spherical rolling robots. Kayacan and Kayacan et al. (Kayacan E. et al., 2013) proposed an adaptive neuro-fuzzy control method for tracking control of the linear motion of a spherical rolling robot. The proposed controller is comprised of a neuro-fuzzy network controller using a sliding-mode-control-theory-based learning algorithm and a conventional controller employed to guarantee the system stability. Zhao and Yu (Zhao and Yu, 2014) presented a decoupled sliding mode control approach for set-point regulation of the linear motion of a spherical rolling robot. A hierarchical structure of the sliding surfaces is adopted for the proposed controller, and the sliding mode control law is developed based on a novel nonlinear reaching law. Yue and Liu et al. (Yue M. et al., 2014) investigated an adaptive sliding mode control scheme for velocity tracking control of the linear motion of a spherical rolling robot. The proposed sliding mode control law is deduced based on the sliding surface with an integral element and the linearized system dynamics, and an adaptive mechanism is adopted to estimate uncertain rolling resistance. Yue and Liu et al. (Yue M. et al., 2014) presented an adaptive hierarchical sliding mode control scheme for tracking control of the linear motion of a spherical rolling robot. The sliding mode control law is derived on the basis of the linearized system dynamics, and an extended state observer is utilized for the estimation of rolling resistance moment. The sliding mode control approaches reported in these studies can deal with the unknown external disturbances well. However, the impact of other uncertainties such as parameter variations and model errors brought by local linearizations in the system dynamics have not been sufficiently considered in these studies.

In this paper, an adaptive decoupled sliding mode control scheme is developed for set-point regulation of the linear motion of a pendulum-driven spherical rolling robot. Different from conventional decoupled sliding mode control approaches (Mahmoodabadi M. J. et al., 2013; Bayramoglu and Komurcugil, 2013; Mahmoodabadi, Mommennejad and Bagheri, 2014), the proposed control scheme adopts a differentiable intermediate variable to design the second-layer sliding surface and utilizes an adaptive algorithm to adjust the switching gain of the decoupled sliding mode controller online. The proposed control scheme guarantees the stability of the overall system well and inherently avoids the problem in conventional designs that the equivalent control law cannot be directly derived. Additionally, the proposed control scheme does not require a priori full knowledge of the system dynamics, and it has strong robustness against parameter variations and external disturbances.

The rest of this paper is structured as follows. In Section 2, the dynamic equations of the linear motion of the spherical rolling robot is derived. In Section 3, the adaptive decoupled sliding mode control law is proposed, and the stability of the closed-loop control system is analysed. In Section 4, simulation results are presented to validate the effectiveness and robustness of the proposed control scheme. Finally, the conclusions are drawn in Section 5.

2. DYNAMIC EQUATIONS OF THE LINEAR MOTION

2.1. System Description of BYQ-XII

BYQ-XII is a novel pendulum-driven spherical rolling robot with three actuator inputs. BYQ-XII mainly consists of a spherical shell, an internal frame and a counter-weight pendulum. The actuation mechanism of BYQ-XII is comprised of three independent actuators: a drive motor, a spin motor, and a tilt motor. Figure 1 illustrates the basic structure of BYQ-XII. A track is arranged at the equator of the inner surface of the spherical shell, and a walking bracket is connected with the track through a traveling wheel which is meshed with the ring rack on the track. The guide wheels and the spin motor are mounted on the walking bracket, and the axle of the spin motor is connected with the traveling wheel. One end of the internal frame is connected with the walking bracket through the ball bearings, and the other end of the internal frame is connected with a rotary bracket through the axle of the drive motor. Also, the rotary bracket is connected with the track through the guide wheels. The tilt motor is located on the internal frame, and its axle is connected with the counter-weight pendulum through the link of the pendulum. Note that the rotational axis of the tilt motor is perpendicular to that of the drive motor.

BYQ-XII can move omnidirectionally on its rolling plane by using the three actuators. When the spin motor is controlled to rotate around its axis, both the internal frame and the counter-weight pendulum rotate relative to the spherical shell around the axis which is perpendicular to the equatorial plane of the spherical shell and passes through the geometric center of the spherical shell. As a result, the heading direction of the spherical rolling robot is changed. In this way, the turning in place motion of the spherical rolling robot is achieved. When the drive motor is controlled to rotate around its axis, the internal frame rotates relative to the spherical shell around the axis of the drive motor. This indirectly makes the counter-weight pendulum swing forward or backward, and then the linear motion of the spherical rolling robot is achieved. The steering motion of the
spherical rolling robot is achieved by using the tilt motor. When the tilt motor is controlled to rotate around its axis, the pendulum can swing from side to side. This allows the spherical rolling robot to tilt on one side or the other as the spherical shell rolls. The novel driving mode of three actuator inputs not only makes the spherical rolling robot move more flexibly than traditional designs, but also endows the robotic system with the fault-tolerant capability. When the tilt motor or the spin motor faults, the remaining two motors can still guarantee the omnidirectional mobility of the spherical rolling robot.

![Figure 1. Basic structure of BYQ-XII](image)

**2.2. Dynamic Modeling**

In derivation of the dynamic equations of the linear motion of the spherical rolling robot, we first make the following assumptions: (1) The outer shell of the spherical rolling robot is a rigid, homogeneous, and thin-walled spherical shell; (2) The spherical shell rolls along a straight-line over a perfectly flat surface without slipping; (3) The center of mass of the internal frame coincides with the geometric center of the spherical shell; (4) The counter-weight pendulum is modelled as a rigid, massless link and a weight at its end, and the axis of the counter-weight pendulum is attached at the geometric center of the spherical shell; (5) Linear viscous friction is assumed to operate between the internal frame and the counter-weight pendulum; (6) The pendulum swing angle is assumed to be kept within the interval of \((-\pi, \pi)\).

![Figure 2. Simplified model of the linear motion of BYQ-XII](image)

The simplified planar model with the side view of the spherical rolling robot is depicted in Figure 2, where an inertial coordinate frame \(\Sigma_0\{X,Y\}\) is anchored to the flat surface and the origin \(O\) represents the initial position of the robot. For clarity, the definitions of the system parameters and variables are introduced as follows. \(\theta\) represents the rolling angle of the spherical shell, \(\phi\) denotes the pendulum swing angle, \(x\) is the robot displacement, \(m_s\) is the mass of the spherical shell, \(m_f\) is the mass of the internal frame, \(m_p\) is the mass of the counter-weight pendulum, \(r\) is the radius of the spherical shell, \(l\) denotes the distance between the geometric center of the spherical shell and the centroid of the counter-weight pendulum, \(\tau\) is the driving torque of the drive motor, \(I_s\) is the moment of inertia of the spherical shell, \(I_f\) is the moment of inertia of the internal frame, and \(I_p\) is the moment of inertia of the counter-weight pendulum.

As shown in Figure 2, the configuration of the robotic system can be uniquely determined by the rolling angle of the spherical shell \(\theta\) and the pendulum swing angle \(\phi\). Therefore, it is reasonable to choose them as the two generalized coordinates

\[
\bar{q} = [\bar{q}_1, \bar{q}_2]^T = [\theta, \phi]^T
\]
Next, we develop the dynamic equations by calculating the Lagrangian \( L = T - P \) of the whole system, where \( T \) and \( P \) are the kinetic energy and potential energy of the whole system respectively. The kinetic energy of the robotic system is given by

\[
T = \frac{1}{2} I_1 \dot{\theta}^2 + \frac{1}{2} I_2 \dot{\phi}^2 + m_p r \dot{\phi} \cos \phi \tag{2}
\]

where \( I_1 = (m_1 + m_r + m_p) r^2 + I_r, \ I_2 = m_p r^2 + I_r + I_p \).

If we choose the potential energy of the system to be zero when the link of the pendulum is held in a horizontal position, then we can obtain the potential energy of the robotic system as

\[
P = -m_p g l \cos \phi \tag{3}
\]

where \( g \) is the gravitational acceleration.

Under the assumption of linear viscous damping between the internal frame and the pendulum, the Rayleigh dissipation function due to the viscous friction is given by

\[
\Phi' = \frac{1}{2} \mu (\dot{\theta} + \dot{\phi})^2 \tag{4}
\]

where \( \mu \) is the damping coefficient associated with the bearings between the internal frame and the pendulum.

Subtracting (3) from (2), we can determine the Lagrangian \( L \) of the robotic system. Then, using the Lagrange’s equation containing dissipation function, the dynamic equations of the linear motion of the spherical robot system is derived as

\[
I_1 \ddot{\theta} + m_p r \dot{\phi} \cos \phi + \mu \dot{\theta} + (\mu - m_p r \dot{\phi}) \dot{\phi} = \tau
\]

\[
m_p r \dot{\phi} + I_2 \ddot{\phi} + \mu (\dot{\theta} + \dot{\phi}) + m_p g l \sin \phi = \tau
\]

It follows from the non-slip condition of the spherical shell that \( \dot{\theta} = x/r \), and the moment of inertia of the spherical shell can be directly computed as \( I_s = 2 m_p r^2 / 3 \). Then, substituting the expressions of \( \theta \) and \( I_s \) into (5), we obtain the following equations of motion

\[
M(q) \ddot{q} + N(q, \dot{q}) = B(q) \tau
\]

where \( M(q) \) is the inertia matrix, \( N(q, \dot{q}) \) is the vector of position and velocity dependent forces, \( B(q) \) is the input transformation matrix, which are given by

\[
M(q) = \begin{bmatrix}
\frac{5}{3} m_1 + m_r + m_p & m_p l \cos \phi \\
m_p l \cos \phi & m_p l^2 + I_r + I_p
\end{bmatrix}
\]

\[
N(q, \dot{q}) = \begin{bmatrix}
\frac{\mu}{r} \dot{x} + \left( \frac{\mu}{r} - m_p l \dot{\phi} \right) \dot{\phi} \\
\frac{\mu}{r} \dot{x} + \mu \dot{\phi} + m_p g l \sin \phi
\end{bmatrix}
\]

\[
B(q) = \begin{bmatrix}
1 & 1 \\
q_1 & q_3
\end{bmatrix}^T
\]

It can be seen from (6) that, a spherical robot system which performs linear motion is an underactuated mechanical system with two degrees of freedom and input coupling.

### 3. ADAPTIVE DECOUPLED SLIDING MODE CONTROL

#### 3.1. Controller Design

To facilitate the subsequent control law development, we rewrite (6) in the following form

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_1(x) + g_1(x) u \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= f_2(x) + g_2(x) u
\end{align*}
\]

where \( x = [x_1, x_2, x_3, x_4]^T = [q_1, \dot{q}_1, q_2, \dot{q}_2]^T \) is the state space vector, \( u = \tau \) is the control input, and the nonlinear functions \( f_1(x), f_2(x), g_1(x), g_2(x) \) are represented by
\[ f_i(x) = -\left[ M^{-1}(q)N(q, \dot{q}) \right], \quad g_i(x) = \left[ M^{-1}(q)B(q) \right], \quad i = 1, 2 \] (8)

where \([A]\) represents the \(i\)-th component of the column vector \(A\).

Consider the robotic system (7), we first decouple the whole system into two subsystems with a second-order canonical form as follows

\[ \begin{align*}
    A: \begin{cases}
    \dot{x}_1 = x_2 \\
    \dot{x}_2 = f_i(x) + g_i(x)u
    \end{cases} \\
    B: \begin{cases}
    \dot{x}_3 = x_4 \\
    \dot{x}_4 = f_i(x) + g_i(x)u
    \end{cases}
\] (9)

We can observe from (6) that, the stable equilibrium state of the robotic system corresponds to the situation in which the link of the pendulum is in the vertical down position (i.e., \(\phi = 0\)). Having this in mind, now the control objective of the overall system is to design the single control input \(u\) such that both the robot displacement \(x_i = x\) and the pendulum swing angle \(x_i = \phi\) converge to their respective desired values.

According to the above analysis, we define the following pair of first-layer sliding surfaces for the two second-order subsystems

\[ s_1 = \dot{e}_x + c_1 e_x, \quad s_2 = \dot{e}_\phi + c_2 \phi \] (10)

where \(c_1\) and \(c_2\) are real positive constants, the robot positioning error \(e_x\) is defined as \(e_x = x - x_d\), and \(x_d\) represents the desired robot position. Without loss of generality, it is assumed that the first and second time derivatives of the desired robot position are bounded.

Note that there is only one control input available on the whole robotic system. To simultaneously stabilize the two subsystems \(A\) and \(B\) through the single control input \(u\), we introduce an intermediate variable \(z\) which is a differentiable function of \(s_2\) as follows

\[ z = Z \cdot \text{saf}(s_2, \varepsilon) \quad \text{saf}(s_2, \varepsilon) = \frac{s_2}{\sqrt{s_2^2 + \varepsilon}} \] (11)

where \(0 < \varepsilon < 1\), and \(\varepsilon\) is a real positive constant.

Incorporating the intermediate variable \(z\) into the first-layer sliding surface \(s_1\), we can obtain the following second-layer sliding surface

\[ S = \dot{e}_x + c_1 (e_x + z) \] (12)

Differentiating (11) with respect to time, we obtain

\[ \dot{z} = \sigma(Z, s_2, \varepsilon) \cdot \dot{s}_2 \quad \sigma(Z, s_2, \varepsilon) = \varepsilon Z \cdot (1 + s_2^2) \frac{1}{2} \] (13)

From (9) and (10), it follows that

\[ \dot{s}_2 = \ddot{\phi} + c_2 \dot{\phi} = f_2 + c_2 x_4 + g_2 u \] (14)

Differentiating (12) with respect to time and using (9), (13) and (14), we obtain

\[ \dot{S} = \dot{e}_x + c_1 (\dot{e}_x + \dot{z}) \\
= f_1 + c_1 \dot{e}_x - \dot{x}_2 + g_1 u + c_1 \sigma \dot{s}_2 \\
= f_1 + c_1 \dot{e}_x + c_1 \sigma (f_2 + c_2 x_4) - \ddot{x}_2 + (g_1 + c_1 \sigma g_2) u \] (15)

Then, the equivalent control law of the second-layer sliding surface can be derived from (15)

\[ u_{eq} = -f_1 + c_1 \dot{e}_x + c_1 \sigma (f_2 + c_2 x_4) - \ddot{x}_2 \\
= g_1 + c_1 \sigma g_2 \] (16)

We adopt the following structure for the decoupled sliding mode controller

\[ u = u_{eq} + u_{sw} \] (17)

where \(u_{sw}\) is the switching control law to be synthesized later.

Substituting (17) into (15), we obtain
\[ \dot{S} = (g_1 + c_1 \sigma g_2) u_{sw} \] (18)

To ensure the stability of the closed-loop system, the switching control law \( u_{sw} \) should be designed in such a way that the sliding condition for the second-layer sliding surface \( S \) is viable. With this in mind, we adopt the following switching control law

\[ u_{sw} = -\xi(t) \cdot \text{sgn}(S) \]

(19)

where \( \xi(t) \) is a positive time-varying switching gain to be defined later.

To eliminate the chattering phenomena of the switching controller (19), the following boundary layer switching control law is adopted for practical implementation

\[ u_{sw} = -\xi(t) \cdot \text{sat}(S) \]

(20)

where the saturation function \( \text{sat}(S) \) is defined as follows

\[ \text{sat}(S) = \begin{cases} \text{sgn}(S), & \text{if } |S| > \delta \\ S/\delta, & \text{if } |S| \leq \delta \end{cases} \]

(21)

where \( \delta \) is the width of the boundary layer.

The time-varying switching gain \( \xi(t) \) is defined as follows.

(1) If \( |S| > \delta \), then \( \xi(t) \) is tuned by the following adaptation law

\[ \dot{\xi}(t) = \text{Proj}(\gamma |S|) \]

(22)

where \( \gamma \) and \( \xi(0) \) are real positive constants, and the projection mapping \( \text{Proj}(\cdot) \) is defined as follows

\[ \text{Proj}(\psi) = \begin{cases} \psi, & \text{if } \xi(t) < \xi_{\text{max}} \\ 0, & \text{if } \xi(t) = \xi_{\text{max}} \end{cases} \]

(23)

where \( \xi_{\text{max}} \) is the specified maximum value of \( \xi(t) \) satisfying \( \xi_{\text{max}} > \xi(0) \).

(2) If \( |S| \leq \delta \), then \( \xi(t) \) is designed as follows

\[ \xi(t) = \lambda |\eta| \]

(24)

where \( \lambda \) is a real positive constant, \( \eta \) is the average value of \( \text{sgn}(S) \) obtained through a first-order low-pass filter

\[ \tau_0 \dot{\eta} + \eta = \text{sgn}(S) \]

(25)

where \( \tau_0 \) is the filter time constant.

Then, the following decoupled sliding mode control law can be derived from (17)

\[ u = -f_1 + c_1 \dot{\xi} + c_1 \sigma (f_2 + c_3 x) - \ddot{\xi} + \xi(t) \cdot \text{sat}(S) \]

(26)

Remark 1: The aim of the proposed adaptive decoupled sliding mode control is to bring the second-layer sliding surface \( S \) to zero. However, under the influence of parameter variations and external disturbances upon the system performance in practical applications, it is impossible for the second-layer sliding surface \( S \) to be exactly zero. In other words, the boundary layer sliding mode control law (26) can only assure that the second-layer sliding surface \( S \) reaches a zero region. Without loss of generality, we assume the second-layer sliding surface \( S \) to be approximately zero when \( |S| \) decreases to \( |S| \leq \delta \).

3.2. Stability Analysis
**Theorem 1:** Consider the spherical robot system (7), define the first-layer sliding surfaces (10) and the second-layer sliding surface (12), and adopt the adaptive decoupled sliding mode law (26). Then, the second-layer sliding surface $S$ converges asymptotically to zero.

**Proof:** To prove this theorem, a Lyapunov function candidate is introduced as follows

$$V(t) = \frac{1}{2} S^2 + \frac{1}{2\gamma} (\xi_{\text{max}} - \xi)^2$$

(27)

Suppose that $|S| > \delta$. Substituting (20) into (18) and using (21), we obtain

$$\dot{S} = -\xi(t) \cdot \text{sgn}(S)$$

(28)

Then, from (27), (28), (22) and (23), it follows that

$$V(t) = S \cdot \dot{S} + \frac{1}{\gamma} (\xi - \xi_{\text{max}}) \xi(t)$$

$$= -\xi|S| + \frac{1}{\gamma} (\xi - \xi_{\text{max}}) \text{Proj}(\gamma|S|)$$

$$= -\xi_{\text{max}} |S|$$

Integrating both sides of (29) with respect to time, we obtain

$$V(t) - V(0) = -\xi_{\text{max}} \int_0^t |S| \, d\rho$$

(30)

From (30), it follows that

$$\xi_{\text{max}} \cdot \lim_{\rho \to \infty} \int_0^\rho |S| \, d\rho \leq V(0) < \infty$$

(31)

From (27) and (30), we further obtain

$$\frac{1}{2} S^2 \leq V(t) \leq V(0) < \infty$$

(32)

From (32), it follows that $S \in L_\infty$. From (22) and (23), it can be inferred that $\xi(0) \leq \xi(t) \leq \xi_{\text{max}}$, which implies that $\xi(t) \in L_\infty$. Then, from (28), it follows that $\dot{S} \in L_\infty$. According to (31), we obtain $S \in L_\infty$. Consequently, application of Babalat’s lemma indicates that the second-layer sliding surface $S$ converges asymptotically to zero, i.e., $\lim_{t \to \infty} S = 0$.

**Theorem 2:** Consider the spherical robot system (7), define the first-layer sliding surfaces (10) and the second-layer sliding surface (12), and adopt the adaptive decoupled sliding mode law (26). Then, the first-layer sliding surfaces $s_1$ and $s_2$ converge asymptotically to zero.

**Proof:** To prove this theorem, a new variable $\hat{z}$ is defined as follows

$$\hat{z} = c_1 \cdot \text{saf}(s_2, \delta)$$

(33)

From (33) and (11), it follows that $|\hat{z}| < c_1 < \infty$, which implies that $\hat{z} \in L_\infty$. According to (10)–(12), and (33), the second-layer sliding surface $S$ can be expressed in the following form

$$S = s_1 + Z \cdot \hat{z}$$

(34)

Differentiating (34) with respect to time, we obtain

$$\dot{S} = \dot{s}_1 + Z \cdot \dot{\hat{z}}$$

(35)

From (9) and (10), it follows that

$$\dot{s}_1 = f_1 + c_1 \epsilon_c - \tilde{x}_d + g_u$$

(36)

Since each term on the right-hand side of (36) is bounded, it implies that $\dot{s}_1$ is also bounded, i.e., $\dot{s}_1 \in L_\infty$. It has been proved that $\dot{S} \in L_\infty$, then it follows from (35) that $\hat{z} \in L_\infty$. 

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Note that the selection of the parameter $Z$ does not affect the stability of the whole system. Then, we can construct the following two second-layer sliding surfaces

$$S_1 = s_1 + Z_1 \cdot \dot{z}$$
$$S_2 = s_2 + Z_2 \cdot \dot{z}$$

where $0 < Z_1 < 1, 0 < Z_2 < 1, Z_1 \neq Z_2$.

Then, it follows from (37) that $S_1 \neq S_2$ if $\dot{z} \neq 0$. Without loss of generality, it is further assumed that

$$0 < \lim_{t \to +\infty} \int_{\rho_0}^{\infty} S_1^2 \, d\rho < \lim_{t \to +\infty} \int_{\rho_0}^{\infty} S_2^2 \, d\rho < \infty$$

From (37) and (38), we can obtain

$$\infty > \lim_{t \to +\infty} \int_{\rho_0}^{\infty} (S_1^2 - S_2^2) \, d\rho$$
$$= \lim_{t \to +\infty} \int_{\rho_0}^{\infty} \left[ (Z_1^2 - Z_2^2) \dot{z}^2 + 2(Z_1 - Z_2) \dot{z} \dot{s}_1 \right] \, d\rho$$
$$= \lim_{t \to +\infty} 2(Z_1 - Z_2) \ddot{s}_1 \dot{\rho} - \lim_{t \to +\infty} \int_{\rho_0}^{\infty} (Z_1 - Z_2)^2 \dot{z}^2 \, d\rho > 0$$

According to (31), we can infer that $S_1 \in L_1$. Then, it follows from (39) that

$$\lim_{t \to +\infty} \int_{\rho_0}^{\infty} \dot{z}^2 \, d\rho < \lim_{t \to +\infty} \int_{\rho_0}^{\infty} \frac{2\ddot{s}_1}{Z_1 - Z_2} \, d\rho$$
$$\leq \frac{2}{|Z_1 - Z_2|} \lim_{t \to +\infty} \frac{\|S_1\|}{\|S_2\|}$$
$$= \frac{2}{|Z_1 - Z_2|} \|S\| < \infty$$

From (40), it follows that $\dot{z} \in L_1$. Note also the facts that $\ddot{z} \in L_\infty$ and $\dddot{z} \in L_\infty$. Thus, application of Babalat’s lemma indicates that $\lim_{t \to +\infty} \dot{z} = 0$. Then, it follows from (33) and (11) that $\lim_{t \to +\infty} s_1 = 0$. Since it has been proved that $\lim_{t \to +\infty} \dot{S} = 0$ and $\lim_{t \to +\infty} \ddot{s} = 0$, it follows from (34) that $\lim_{t \to +\infty} s_1 = 0$.

4. NUMERICAL SIMULATION VERIFICATION

To verify the effectiveness of the proposed adaptive decoupled sliding mode control scheme, some numerical simulation tests are performed using MATLAB. The entire numerical simulation process includes three parts. In the first part, the system performance of the proposed control scheme is checked. In the second part, the tolerance of the designed control system to parameter variations is tested. In the third part, the robustness of the closed-loop control system against different kinds of external disturbances is examined.

4.1. Control Performance Test

In this simulation, the control performance of the proposed adaptive decoupled sliding mode control scheme is verified. The nominal parameters of the spherical robot system are decided as

$$m_r = 3 \text{ kg} \quad m_f = 2.15 \text{ kg} \quad m_p = 2.85 \text{ kg} \quad r = 0.175 \text{ m} \quad l = 0.094 \text{ m}$$
$$I_f = 0.021 \text{ kg} \cdot \text{m}^2 \quad I_p = 0.0011 \text{ kg} \cdot \text{m}^2 \quad \mu = 0.02 \text{ N} \cdot \text{m} \cdot \text{(rad/s)}^{-1}$$

The gravitational acceleration is determined as $g = 9.81 \text{m/s}^2$. The initial conditions of the robotic system are set as $x_0 = 0, \dot{x}_0 = 0, \phi_0 = 0, \dot{\phi}_0 = 0$. The target position of the robot is set as $x_g = 1 \text{ m}$. The parameters of the sliding surfaces are selected as $c_1 = 1.8, c_2 = 5.3, c = 1, Z = 0.7$. The parameters of the control law are selected as $\xi(0) = 0.8, \gamma = 7.6, \xi_{\text{max}} = 5.3, \delta = 0.1, \lambda = 0.1, \tau_0 = 0.02$.

The control performance of the proposed control scheme is illustrated in Figure 3. It is found that both the robot displacement and the pendulum swing angle are stabilized to the desired values. Specifically, the spherical rolling robot accurately reaches its destination with no overshoot at 3.9s, and soon after the robot stops the pendulum swing angle converges to zero at 4.8s. Additionally, during the whole control process the maximum pendulum swing angle is 0.48 rad. The simulation results not only demonstrate the validity of the proposed...
control scheme, but also show that the proposed control scheme can provide excellent transient and steady-state performances.

**Figure 3.** Simulation results of the control performance test

### 4.2. Parameter Variation Robustness Test

In this simulation, the robust performance of the proposed control scheme against parameter variations is validated. To this end, we set the system parameters in (41) to have ±10% discrepancy from their nominal values. However, the changes in the system parameters are not accounted for in the controller, and the proposed control law (26) is still designed by using the nominal values of the system parameters. Additionally, the controller parameters remain the same as in the first simulation test.

**Figure 4.** Simulation results of the parameter variation robustness test

The simulation results of parameter variation robustness test are illustrated in Figure 4. It is evident that, even in the presence of significant parameter variations, the proposed control scheme can still achieve satisfactory control performance. Although there are some changes in the control performance due to severe parameter variations, the proposed control scheme still guarantees the stability of the overall system well. The simulation results illustrate the robustness property of the closed-loop control system against parameter variations and demonstrate the model-free advantage of the proposed control scheme.

### 4.3. External Disturbance Robustness Test

In this simulation, the robust performance of the proposed control scheme against external disturbances is further validated. To this end, three different kinds of disturbance signals are applied to the spherical shell during the control process. An impulse disturbance signal is injected between 7s and 10s, a sine wave disturbance signal is introduced between 15s and 18s, and a random disturbance signal is added between 23s and 26s. All the disturbance signals have the same maximum amplitude of 0.05m. Additionally, the controller parameters remain the same as in the first simulation test.

**Figure 5.** Simulation results of the external disturbance robustness test
The simulation results of external disturbances robustness test are illustrated in Figure 5. It is clear that, even in the presence of various extraneous disturbances, the proposed control scheme can still provide satisfactory control performance. Especially, the effects of different kinds of disturbance signals can be quickly suppressed and eliminated. The simulation results directly validate the strong disturbance rejection capability of the proposed control scheme.

5. CONCLUSIONS

In this paper, the regulation control problem of the linear motion of an underactuated spherical rolling robot is discussed. A novel robust control approach based on an adaptive decoupled sliding mode control algorithm is proposed. The proposed control approach can dynamically adjust the switching gain of the controller according to the variations of the system states, and it allows the decoupled sliding mode controller to better adapt to the system uncertainties. The switching gain of the controller increases gradually in reaching phase to accelerate the attraction of the sliding surface, and the equivalent-control-dependent switching gain is adopted in sliding mode to reduce the chattering. The hierarchical sliding surfaces of the control system are proven to be asymptotically stable by applying Lyapunov method and Babalat’s lemma. The simulation results not only verify the correctness of the theoretical analysis, but also validate the robustness performance of the proposed control approach. In the future work, the proposed control approach will be experimentally tested on a prototype of the spherical rolling robot, and the improvement of the proposed control approach to achieve better performance is the focus of our future work.

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