Profit Allocation of Supply Chain Alliance Based on Experiment

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Abstract
The concept of integrated supply chain alliance management highlights the leveraged benefits of firms collaborating to achieve common goals. To create collaboration among firms, one of the most important steps is to establish a fair and reasonable profit allocation contract. The profit allocation methods commonly used are the concepts of coalitional game such as the nucleolus and Shapley value. This paper conducted a profit sharing experiment by simulating the scenario of a three-party coalitional game. The experiment result is used to compare with the theoretical solutions in practicality and input orientation. The results show that the input factor, quantified as the resource devotion to the grand alliance influences significantly on the expected profit share of the players. A fair and reasonable profit allocation scheme of supply chain alliance should take the players’ input and other influence factors into consideration.

Keywords: Supply chain alliance, Profit allocation, Shapley value, the core, Experiment economics.

1. INTRODUCTION
Driven by increasing competitive pressure on the global scale, enterprises with common goals establish collaborations to beat the competition. Each cooperative entity has its core competency in specified area such as product design, manufacturing and sale distribution. Supply chain alliances bring together independent entities’ core advantages to develop innovative products, optimize logistic activities, reduce operation cost and improve service quality. Enterprises will join a coalition and cooperate only when they can gain greater benefits than those gained individually. So a reasonable and fair allocation plan is critical to the long-term stability of the supply chain alliance.

The implementation and stability of supply chain alliance has been studied by many researchers from different perspectives. Game theory is frequently used to solve the problem of profit sharing among collaboration members. The mostly used solution concepts for coalition profit sharing are Shapley value (Shapley and Shubik, 1954), core (Aumann and Maschler, 1964) and nucleolus. The Shapley value has been revised and widely applied to solve the profit distribution problems in different scenarios of coalitions. Many studies concentrated on the subjective uncertainty regarding the evaluation of coalition payoffs. For example, applications of fuzzy set theory to deal with the incomplete information by Li and Zhou et al. (2015); application of uncertain Shapley value to solve the profit allocation problem by Gao and Yang et al. (2017) and so on. Profit allocation scheme of supply chain alliance, regarding many subjective and objective influence factors is difficult to be solved perfectly by a single theoretical model with consideration covering every aspect. The analysis of game theoretical models in the laboratory is another important line of research in this problem. Since Vernon Smith was awarded Nobel Prize in 2002 (2016), experimental economics was given a considerable boost and now there has been hundreds of experiments studying the problems of strategic interaction between multiple players. Katok and Pavlov (2013) find that a theoretical contract may not coordinate the supply chain when the information about the players’ preferences for fairness is incomplete. Chow and Wang et al. (2015) suggested by their experiment results that there is an inverse U-shaped relationship between the supply chain profit and the minimum profit share ratio. Most previous studies solved the profit allocation problem either by using developed models of Game theory or experiments and simulations without comparing between the solutions of the two approaches.

In this paper, we conducted a laboratory experiment to investigate in the players’ expectations to their profit share in the grand alliance. A three-party coalition with subset coalition payoffs and quantified resource devotion was assumed. The experiment solution was collected from the bar gains of the participants. Then we used the classic and developed models of Game theory to solve the same profit allocation problem. The empirical data of the experiment result can be used to analyze the theoretical solutions so as to assess the practicality and fairness of the solutions.

2. ANALYTICAL BACKGROUND
In this section, we first introduce the experiment background by setting a scenario in which three enterprises specializing in different areas aiming to form a coalition and maximize their respective expected profits in the grand coalition. Then we recall the theories of coalitional game which will be used later to solve the allocation problem in the scenario.
2.1. Introduction to the background

There are three enterprises named respectively A, B and C intending to establish a grand cooperative alliance. Each enterprise has its core advantages in a respective area as follows. A is a product developer concentrating on the technical innovation; B is a product manufacturer equipped with rapid and mass production capacity; C is a product distributor with mature distribution network occupying considerable market share. The three cooperative partners know each other well and are willing to devote their core competencies to achieve complementary advantages, common risk and profit sharing for the grand alliance.

A has developed a patented technique of product M. Based on the patent, A, B and C can form an supply chain alliance and perform the design, manufacture and retail of product M to make profit. The profits A, B and C can make if it does not join the coalition are a, b and c respectively. If each two of the enterprises form a two-party coalition, the profits are d for A and B; e for A and C; f for B and C. The profit that the three-party grand coalition can make is g.

To persuade the three enterprises to form the grand alliance, the profit shared by each player must be higher than that they gained individually or shared in a two-party coalition. If not, at least one enterprise will not join the three-party coalition and the cooperation will be failed.

The prerequisites of the grand coalition’s formation in above scenario are as follows.
1) The payoffs of any two-player coalitions are higher than the sums of payoffs individual players gain respectively, specified as follows.
   \[ d > a + b, \quad c > b + c, \quad f > a + c \]
2) The payoff of the grand coalition is higher than the sums of any two-player coalitions’ payoffs and that of the third players, specified as follows.
   \[ g > d + c, \quad g > e + a, \quad g > f + b \]

2.2. Theories of coalitional game

The enterprises with common goals join the supply chain alliance like the players join the game. In a coalitional game, when the players cooperate to form alliances, different plays will gain different profits in different alliances. When one player obtains its maximum profit in a particular coalition, the other ones may not be satisfied with the profit they shared. The unsatisfied partners may not fully devote their resources and competencies to the alliance so that the profit or efficiency of the alliance may be influenced.

Studies in coalitional game theory focuses on two major issues: coalition formation and distribution of benefit gained through cooperation. If the unities can obtain more benefit through cooperating together than before, they will devote their resource and competencies to form a coalition rather than to play the game individually. This is the first precondition that the cooperation can be achieved. When the partners try to join in the game, they will forecast how much share of the benefit can be obtained in advance rather than bargaining for their share after the game. Therefore, a predetermined benefit distribution contract with a reasonable and satisfactory allocation scheme is essential for the coalition’s efficiency and stability. Here we introduce the concepts of the Shapley value as a parallel solution comparing to the experiment and the core as the results test.

A. The Shapley value

Shapley value is introduced by Shapley L. S. [1] in early 1950s to fairly distribute the payoffs of a set of players in a coalitional game.

A coalitional game can be denoted as \( \Gamma = [N, v] \) with transferable payoff consists of a finite set \( N \) and a function \( v \) that associates a real number \( v(S) \). In which \( N = \{1, 2, \ldots, n\} \) is the set of players, \( S \) is every nonempty subset coalition of \( N \) and \( v(S) \) is the payoff of \( S \).

Denote \( X = \{x_1, x_2, \ldots, x_n\} \) as the set of the players’ profit shared from the maximum payoff \( v(N) \). The grand coalition will succeed when \( v(S) \) and \( x_i \) meet the conditions as follows.

\[
\begin{align*}
  v(\emptyset) &= 0 \\
  v(N) &\geq \sum_i v(i) \\
  \sum_i x_i &= v(N) \\
  x_i &\geq v(i), i = 1, 2, \ldots, n
\end{align*}
\]

(1)

(2)

The payoff of player \( i \) in a coalitional game \([N, v]\) is the Shapley value, defined as \( \varphi_i(v) = (\varphi_1(v), \varphi_2(v), \ldots, \varphi_n(v)) \) and is calculated by the condition
where $s$ is the number of players in the coalition $S$ and $\Delta_i(S) = v(S) - v(S \setminus i)$ is the marginal contribution of player $i$ to coalition $S$.

B. The core

The core of a coalitional game is the set of all stable actions of the grand coalition that no coalition can break away and select another action that all its players prefer. The core of a game always exists because it is defined as a set of actions, though the game may be the empty set. When the core can be alternatively defined as the set of all actions of the grand coalition upon which no coalition can improve, it can be used to examine whether an action is efficient.

The core of a game $v$ satisfies the following conditions:

$$\sum_{i \in S} x_i \geq v(S)$$
$$\sum_{i \in N} x_i = v(N)$$

where $x_i$ is the imputation of player $i$ and every imputation in the core of the coalitional game is an efficient allocation.

3. EXPERIMENTAL SOLUTION

Although conceptions of coalitional game can solve the profit distribution problem of supply chain alliances, the fairness of the solutions is controversial. Enterprises establish an alliance is like players join a coalitional game, but the game cannot perfectly simulate the alliance environment. The profit allocation scheme of the cooperation contract is signed basically according to the enterprises’ payoffs while other factors should also be taken into consideration, for example the players’ input to the alliance, expectations to the profit share, position importance and negotiation abilities. Moreover, coalitional game theories such as Shapley value and the nucleolus usually neglect the influence of player’s resource devotion. But a player evaluates an ongoing relationship by assessing his or her inputs into and returns from the cooperation, relative to that the other players contribute to and receive.

To examine the fairness and practical of the theoretical solutions, here we design an experiment to solve the profit allocation problem in scenario 2.1 and compare with the solutions. When running an experiment, the first basic aspect is that the participants in the laboratory must face a concrete problem. To ensure the experiment background specific, firstly we bring in a case of supply chain alliance with numbers of payoffs and quantified core competencies.

The scenario of 2.1 is specified as follows.

3.1. Experiment background

Three enterprises A, B and C are willing to form a supply chain alliance in the vacuum cup industry in order to maximize their profits. In the experiment we quantify their core competencies as their resource devotion to the alliance. The payoff $v(S)$ of the grand coalition’s subset coalitions ($S$) and quantified resource devotions are as follows.

<table>
<thead>
<tr>
<th>$S$</th>
<th>A, B</th>
<th>A, C</th>
<th>B, C</th>
<th>A, B, C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v(S)$</td>
<td>2</td>
<td>3</td>
<td>10</td>
<td>7</td>
</tr>
</tbody>
</table>

Notes. Subset coalition with players A, B and C is the grand coalition $N$.

<table>
<thead>
<tr>
<th>Player $i$</th>
<th>A(Designer)</th>
<th>B(Manufacturer)</th>
<th>C(Distributor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core competencies</td>
<td>Product design</td>
<td>Manufacturing</td>
<td>Product marketing</td>
</tr>
<tr>
<td>Quantified resource devotion $R(i)$</td>
<td>5</td>
<td>10</td>
<td>2</td>
</tr>
</tbody>
</table>

Notes. Numbers are points in the experiment.
A is specialized in the development and design of vacuum cups. The company is always committed to the development and utilization of new technologies. It invests capital to hire experts and import advanced equipments. We quantify the resource devotion of A as 5; here we neglect the method of input quantification.

B is a 30 years’ old manufacturer specialized in the production of stainless steel products. The enterprise imports high quality stainless steel from Germany and equips the most advanced product line. The resource devotion of B is quantified as 10.

C is a national chain retail enterprise with chain stores in over 10 cities across the country. The enterprise’s sale network can efficiently support the distribution of the products B produces. We quantify its resource devotion as 2.

3.2. Laboratory setting

The purpose of the experiment is to simulate the decision making process of three partners bargaining for their profit share subject to the grand coalition. We recruited six participants with 3 to 5 years management experience working in different companies. The participants play the roles of three players A, B and C in the scenario. The background above was introduced to them to make sure there is no information asymmetry. The decision making process was repeated 20 times and roles of players was assigned randomly by taking draws before each round. This regulation is set to insulate the influence of bias against any industry area or moral hazard from unequal status. The decision process is round based so the participants can learn the coalition partners’ strategies and revise his or her decision in each round. To incert the participants act like the players maximizing their profit, a fund of $200 as the payoff of the grand coalition was paid to them in accordance with their profit gained in the experiment.

The varieties used in the experiment are set as follows.

The three-party coitalional game is denoted as $\Gamma = [N, V]$, $N = \{A, B, C\}$. Denote $S$ as the subset coalitions of $N$; $V(S)$ as the payoff (profit) of $S$; $y_{ij}$ as the bargain of player $i$ in round $j$; $y_{ij}$ as the valid bargain of player $i$ in round $j$; $h_{ij}$ as the point participant $k$ gained in round $j$, where $i = A, B, C$; $j = 1, 2, ..., 20$; $k = 1, 2, ..., 6$.

The value of $h_{ij}$ was calculated by the following model.

$$h_{ij} = \begin{cases} 
\alpha - |x_{ij} - E(i)| & \text{$x_{ij}$ is valid (meet all the upper limits of coalitions)} \\
(\alpha - |x_{ij} - E(i)|) / 2 & \text{$x_{ij}$ is valid (meet one upper limit of two-party coalitions)} \\
0 & \text{$x_{ij}$ is invalid (does not meet any upper limits)}
\end{cases}$$

(6)

where $n$ is the number of times that player $x_{ij}$ is valid; $\alpha$ is the basic index of players’ accumulated points set as the maximum absolute value of the difference between $y_{ij}$ and $E(i)$; $E(i)$ is the expected value of player $i$’s valid bargains; $h_{k}$ is the sum of participant $k$’s accumulated points in 20 rounds. The values of $\alpha$, $E(i)$ and $h_{k}$ are calculated as follows.

$$h_{k} = \sum_{j=1}^{20} h_{ij} ; \quad \alpha = \max_{i,j} |y_{ij} - E(i)| ; \quad E(i) = \sum_{j=1}^{20} y_{ij} / n$$

The experiment focuses on players’ evaluation of profit share in the grand coalition according to each player’s input and output in all subset coalitions. So we neglect the negotiation process between players so as to clean the noise of some influence factors occurred during the negotiation. Without negotiation, the sum of the players’ bargains cannot equal to the payoff of the grand coalition exactly. So we set the upper limits to solve this problem. Whether the grand coalition can be formed depends on the relationship between players’ bargains and the upper limit ($\lim(S)$) for every subset coalition.

$\lim(N)$ floats a bit from $\nu(N)$ to 16. The payoff of coalition consists A and B is 10, while the payoff of the grand coalition is 14. Player C expects his or her profit at least higher than the payoff he plays independently. So we set the upper limit of subset coalition $\{A, B\}$ as:

$$\lim(A, B) = [10 + (14 - 1)] / 2 = 12$$

We set the upper limits of other two-party coalitions in the same way:

$$\lim(A, C) = [5 + (14 - 3)] / 2 = 8$$

$$\lim(B, C) = [7 + (14 - 2)] / 2 = 10$$
Table 3. Subset coalitions’ upper limits of bargains

<table>
<thead>
<tr>
<th>Subset coalition</th>
<th>A, B</th>
<th>A, C</th>
<th>B, C</th>
<th>A, B, C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper limit (\lim(S))</td>
<td>12</td>
<td>8</td>
<td>10</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 3 was shown clearly in front of the participants, reminding them about the risk of point loss if fail to form the coalitions. According to Table 3, take the bargain of participant 1 as player A in round \(j\) \((x_{Aj})\) for example.

If \(x_{Aj}\) meets the conditions:

\[x_{Aj} + x_{Bj} + x_{Cj} \leq \lim(N)\]  
\[x_{Aj} + x_{Bj} \leq \lim(A, B)\]  
\[x_{Aj} + x_{Cj} \leq \lim(A, C)\]

at the same time, then \(x_{Aj}\) is regarded valid. The corresponding point participant 1 gains in round \(j\) equals to \(h_{Aj} = \alpha - |x_{Aj} - E(A)|\).

If \(x_{Aj}\) only meets one of the following two conditions:

\[x_{Aj} + x_{Bj} \leq \lim(A, B)\]  
\[x_{Aj} + x_{Cj} \leq \lim(A, C)\]

which means player A can only form one coalition with player B or C; \(x_{Aj}\) is regarded invalid. The corresponding point participant 1 gains in round \(j\) equals to

\[h_{Aj} = \left(\alpha - |x_{Aj} - E(A)|\right) / 2\].

If \(x_{Aj}\) does not meet any of the above conditions, \(x_{Aj}\) is regarded invalid and participant 1 gains nothing from this round so that \(h_{Aj} = 0\).

\[E(A) = \frac{\sum_{j=1}^{n} y_{Aj}}{n}\]  
where \(E(A)\) is player A’s expected value of bargains in 20 rounds. After the decision process finished, we would assess the sum of all the participants’ accumulated points and participant \(k\) ’s final payment would be

\[\$200 \times \frac{\sum_{k=1}^{6} h_k}{\sum_{k=1}^{6} h_k} = \frac{\sum_{j=1}^{20} h_{Aj}}{}\] \(\text{(7)}\)

3.3. Experiment process

The data produced in the experiment was recorded by paper and pens. The following tools were prepared before the experiment began.

1) 6 drawing cards with A, B and C written on every two of them;
2) 120 bargain cards for participants to record their bargain during the decision making process;
3) 1 record table to record participants’ accumulated points and calculate their payments.

The experiment was conducted as the following flows.

![Figure 1. Experiment process](image)

Step 1. The experimenter announced the instructions including the purpose, background, regulations and process of the experiment;
Step 2. Number the participants with 1 to 6 and ask them to remember their own number;
Step 3. Announce round 1 begin. Ask every participant to take a draw and decide his or her role of the player;
Step 4. Every two participants with the same role discuss and make their decision together. Remind the participants to record clearly on the decision card;

Step 5. Collect the decision cards and announce the result according to the bargains. The result is about whether the grand coalition or subset coalitions succeed in this round;

Step 6. Repeat the decision process 20 times;

Step 7. After finish the decision process, calculate every participant’s payment according to the regulation;

Step 8. Pay the payment to every participant, and interview them about the feelings of the experiment;

Step 9. Announce the experiment is over and thank the participants for their time. Implement the data process.

3.4. Experiment result

The empirical data of the experiment has been saved and analyzed in “Microsoft Excel” files. There are 120 bargains collected by the decision cards during the experiment. The experimenter processed the bargains in the record card (see appendix 1) and calculated $E(i)$ according to the data. The bargains in round 4 were deleted because every bargain made by player A, B or C is invalid. The values of variables are assessed as follows according to the data from 19 rounds.

$$n_d = n_b = n_c = 19$$

$$E(A) = \frac{\sum_{j=1}^{n} y_{ij}}{n} = 4.74$$

$$E(B) = \frac{\sum_{j=1}^{n} y_{ij}}{n} = 7.95$$

$$E(C) = \frac{\sum_{j=1}^{n} y_{ij}}{n} = 2.16$$

The valid data are shown in table 4 below.

<table>
<thead>
<tr>
<th>$y_{ij}$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>?</td>
<td>10</td>
<td>8</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Notes. Invalid data from round 4 has been deleted

The payment of experiment was calculated by participants’ accumulated points [8]. Accumulated points each round were assessed according to the bargain record table and recorded as in appendix 2. The accumulated index $\alpha$ equals to $\max \{ y_{ij} - E(i) \} = 2.05$. Variables regarding the participants’ payments are calculated and shown in the following table.

<table>
<thead>
<tr>
<th>participant</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_i$</td>
<td>33.43</td>
<td>31.12</td>
<td>27.76</td>
<td>33.44</td>
<td>34.62</td>
<td>32.79</td>
</tr>
<tr>
<td>$h_i / \sum h_i$ (%)</td>
<td>17.31%</td>
<td>16.11%</td>
<td>14.37%</td>
<td>17.31%</td>
<td>17.92%</td>
<td>16.98%</td>
</tr>
<tr>
<td>Payment ($)</td>
<td>34.62</td>
<td>32.22</td>
<td>28.74</td>
<td>34.62</td>
<td>35.84</td>
<td>33.96</td>
</tr>
</tbody>
</table>

where $\sum h_i = 191.87$ and the total payment was $200 and each participant’s payment was calculated by equation (7).

The experiment lasted two hours and was successfully completed. The participants knew the basic knowledge of game theory and have gained management experience from their work. 6 participants have gained their payments respectively according to the calculation result after the experiment.

Due to the limit of laboratory space and fund, we only recruited 6 participants to make decisions in 20 rounds. After about 15 rounds, almost every participant has learnt each other’s risk-return preference and started to get tired. As a result, we found the bargains after round 15 almost stayed the same as in round 15. To clear this noise, we only picked the data from round 1 to 15 for analysis. Excluding the invalid bargains from round 4, data used for analysis are as follows.

<table>
<thead>
<tr>
<th>$y_{ij}$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>(\sum y_{ij})</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>66</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>7</td>
<td>10</td>
<td>8</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>112</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>30</td>
</tr>
</tbody>
</table>
According to Table 6, $E(i)$ was revised to be $\bar{E(i)}$.

$E(A) = 4.71$  $E(B) = 8.00$  $E(C) = 2.14$  $\sum \bar{E(i)} = 14.86$

Adjust to percentage as follows.

$E'(A) = \frac{4.71}{14.86} \times 14 = 4.44$  $E'(B) = \frac{8.00}{14.86} \times 14 = 7.54$  $E'(C) = \frac{2.14}{14.86} \times 14 = 2.02$

The allocation solution of the experiment is $\frac{1}{14}(4.44, 7.54, 2.02) = (31.73\%, 53.85\%, 14.42\%)$, denoted as solution E.

Adjust all the bargains of player A, B and C in 14 rounds to percentage as in the following table.

Table 7. Percentage adjusted bargains

<table>
<thead>
<tr>
<th>j</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>26.6</td>
<td>37.5</td>
<td>25.0</td>
<td>28.5</td>
<td>35.7</td>
<td>28.5</td>
<td>37.5</td>
<td>33.3</td>
<td>33.3</td>
<td>28.5</td>
<td>33.3</td>
<td>33.3</td>
<td>33.3</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>53.3</td>
<td>43.7</td>
<td>62.5</td>
<td>57.1</td>
<td>50.0</td>
<td>57.1</td>
<td>57.1</td>
<td>50.0</td>
<td>53.3</td>
<td>57.1</td>
<td>53.3</td>
<td>53.3</td>
<td>53.3</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>20.0</td>
<td>18.7</td>
<td>12.5</td>
<td>14.2</td>
<td>14.2</td>
<td>14.2</td>
<td>12.5</td>
<td>13.3</td>
<td>13.3</td>
<td>14.2</td>
<td>13.3</td>
<td>13.3</td>
<td>13.3</td>
<td></td>
</tr>
</tbody>
</table>

To investigate the participants’ psychological changes, we analyze the floating situation by drawing the curve graph of 3 players’ bargains.

![Figure 2. Bargains (percentage) of player A, B and C](image)

We can learn from fig 2 that the bargains of player A, B and C are tending to a certain value as the experiment carried on. The bargains fluctuate significantly in the first 8 rounds, then after 12 rounds the bargains tend to be stable. This floating trend illustrates that while the game repeated; the participants gained experience from the previous rounds and finally reached an agreement on the profit allocation solution.

4. SOLUTIONS BASED ON THE COALITIONAL GAME CONCEPTS

According to the experiment scenario set in Tab 1 and Tab 2, apply Shapley value model to solve the profit allocation problem. We introduce two theoretical solutions based on the Shapley value. The 1st one only considers the payoffs of subset coalitions, denoted as Solution 1; and the 2nd one consider both the subset coalitions’ payoffs and the players’ resource devotion, denoted as Solution 2.

4.1. Solution 1

Substitute numbers in Table 1 into equation (3), and the solution is calculated as follows.

$\varphi_A(v) = 4.83$  $\varphi_B(v) = 6.33$  $\varphi_C(v) = 2.83$

The profit share of three players in solution 2 is: $\frac{1}{14}(4.83, 6.83, 2.83) = (34.52\%, 45.24\%, 20.24\%)$

4.2. Solution 2

Solution 2 neglected the input of the players, which was quantified as resource devotions in Table 2. In reality the players evaluate and expect their profit share by comparing his or her own input and output with
others’. Solution 2 is not practical from this point of view. In solution 3 we combine the input and output factor by simply adding them together to indicate the players’ resource devotion. The transferred payoffs of subset coalitions refer to Table 8.

Table 8. transferred payoffs based on the resource devotion

<table>
<thead>
<tr>
<th>S</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>A, B</th>
<th>A, C</th>
<th>B, C</th>
<th>A, B, C</th>
</tr>
</thead>
<tbody>
<tr>
<td>v(S)</td>
<td>7</td>
<td>13</td>
<td>3</td>
<td>25</td>
<td>12</td>
<td>19</td>
<td>31</td>
</tr>
</tbody>
</table>

Substitute numbers in Table 8 into equation (3), and the solution is calculated as follows.

\[
\phi_A'(v) = 9.83 \quad \phi_B'(v) = 16.33 \quad \phi_C'(v) = 4.83
\]

The profit share of three players in solution 3 is: \(\frac{1}{31}(9.83, 16.33, 4.83) = (31.72\%, 52.69\%, 15.59\%)\)

5. TEST AND COMPARISON OF SOLUTIONS

5.1. Test on the solutions by the core

Because the core is defined as a set of stable actions that no other coalition is preferred by any of the players, we can test the validity of the solutions above by examining whether the actions are in the core. Substitute the numbers in table 1 into equation (4) and (5) to solve the core.

A. (0, 1) Standardization

\[
v(N) = \sum_{i \in N} v(i) = 14 - 2 - 3 - 1 = 8 \quad c = \frac{1}{v(N) - \sum_{i \in N} v(i)} = \frac{1}{8}
\]

\[
a_A = -cv(A) = -\frac{1}{4}, \quad a_B = -cv(B) = -\frac{3}{8}, \quad a_C = -cv(C) = -\frac{1}{8}
\]

Derived from \(v'(S) = cv(S) + \sum_{i \in S} a_i \forall S \subseteq N\),

\[|S| = 1: \quad v'(A) = cv(A) + a_A = 0 \quad v'(B) = v'(C) = 0\]

\[|S| = 2: \quad v'(A, B) = cv(A, B) + a_A + a_B = \frac{1}{8} 	imes 10 - \frac{1}{4} - \frac{3}{8} = \frac{5}{8}\]

\[v'(A, C) = cv(A, C) + a_A + a_C = \frac{1}{8} \times 5 - \frac{3}{8} = \frac{1}{4}\]

\[v'(B, C) = cv(B, C) + a_B + a_C = \frac{1}{8} \times 7 - \frac{3}{8} = \frac{3}{8}\]

\[|S| = 3: \quad v'(A, B, C) = cv(A, B, C) + a_A + a_B + a_C = \frac{1}{8} \times 14 - \frac{7}{4} - \frac{3}{8} - \frac{1}{8} = 1\]

B. Solve the core

\(X = (x_A, x_B, x_C) \in E(v)\) is calculated as follows.

\[
x_A \geq v'(A) \quad x_B \geq v'(B) \quad x_C \geq v'(C)
\]

\[
x_A + x_B \geq v'(A, B) \quad x_A + x_C \geq v'(A, C) \quad x_B + x_C \geq v'(B, C)
\]

\[
x_A + x_B + x_C = 1
\]

\[
x_A \geq 0 \quad x_B \geq 0 \quad x_C \geq 0 \Rightarrow 0 \leq x_A \leq 5/8 \quad 0 \leq x_B \leq 3/4 \quad 0 \leq x_C \leq 3/8
\]

The core is the interval of \(x_A \leq 62.5\%, \quad x_B \leq 75\% \quad \text{and} \quad x_C \leq 37.5\%\) which can be demonstrated by the dark area in fig 4. Allocation solutions falling into the area are regarded as valid.
Figure 3. The core of the grand coalition

Display Solution 1, 2 and E by pie charts and compare with the core.

Figure 4. Three solutions pie charts

It is obviously illustrated that Solution 1, 2 and E all meet the upper limit of the core. Every solution is valid. According to Fig 5, the rough shape of Solution E is close to Solution 2. We will compare the solutions further in the next section.

5.2 Comparison between solutions

Although every solution is valid theoretically tested by the core, many influence factors other than the payoffs of subset coalitions are considered when making a real profit allocation decision. Among which the most important factor should be the input of every player including the quantifiable resource devotion and non-quantifiable factors such as the time saved, risk shared and the intellectual capital devoted to the alliance. Some solutions proposed by the concepts of coalitional game are actually not practical.

A. Practicality

In this section, we assess the practicality of two theoretical solutions by comparing them with solution E. Denote $x_i^m$ as the profit share of player $i$ in Solution $m$, where $i = A, B, C$ and $m = 1, 2, E$.

The deviation of Solution 1 and 2 from Solution E can be calculated as in equation (8).

$$SD(m, E) = \sum_{i=A,B,C} (x_i^m - x_i^E)^2$$  \hspace{1cm} (8)

Substitute numbers in table 9 into equation (8) and get $SD(1, E) = 0.011579$; $SD(2, E) = 0.000271$. The results illustrate that $SD(1, E)$ is much higher than $SD(2, E)$, which means Solution 2 with a consideration on the resource devotion is much closer to the practice. According to the interview to the participants after the experiment, almost every participant took the input factor into consideration. Two participants even regarded input factor more important than the output. This explains the far deviation of Solution 1 from the experiment.

B. Input orientation

We then assess the degrees of attention paid to the input by the three solutions by comparing them with the resource devotion. We learnt from Table 2 that the quantified resource devotions $R(i)$ are $R(A) = 5$, $R(B) = 10$, $R(C) = 2$. Standardize $R(i)$ to $(0, 1)$.

$R(A) = 29.41\%$, $R(B) = 58.82\%$, $R(C) = 11.76\%$

The deviation of Solutions from $R(i)$ can be calculated as in equation (9).

$$SD'(m, R) = \sum_{i=A,B,C} [x_i^m - R(i)]^2$$  \hspace{1cm} (9)
Table 9. Standard deviations of solutions from resource devotions

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>SD(m,R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution 1</td>
<td>34.52%</td>
<td>45.24%</td>
<td>20.24%</td>
<td>0.02824389</td>
</tr>
<tr>
<td>Solution 2</td>
<td>31.72%</td>
<td>52.69%</td>
<td>15.59%</td>
<td>0.00575819</td>
</tr>
<tr>
<td>Solution E</td>
<td>31.73%</td>
<td>53.85%</td>
<td>14.42%</td>
<td>0.00371589</td>
</tr>
</tbody>
</table>

It is illustrated in table 9 that SD(1,R) > SD(2,R) > SD(3,R), which means the experiment solution considers the input factor most comparing to the theoretical solutions. When making the allocation decisions, the participants attach importance to the resource devotion factor. The importance degree is even higher than the consideration in Solution 2.

The two comparison results demonstrate that the solutions we commonly adopted by traditional concepts of coalitional game may be far from practice. If player’s payoffs are directed related to its resource devotion, say in liner relationship, this deviation would be minor. However in reality, the relationship between profitability and resource devotion in many enterprises is not obvious. As the relativity weakens, the distance from the reality may increase.

According to the comparison, Solution 2 is closer to the experiment result than Solution 1. If available, the enterprises can make their profit allocation decision based on the experiment result before forming the alliance. The decision process illustrates the players’ real expectations to some extent. An allocation solution adjusted from the experiment result would be more fair and reasonable than theoretical solutions. If experiment is not available, players should take resource devotion into consideration when making allocation decisions.

The concepts of coalitional game have been regarded ideal solutions to the profit allocation problem of supply chain alliance. The theories of Shapley value and the nucleolus solve the allocation problem based on the payoffs of all the subset coalitions, while neglecting other factors influence the decision making process. This results in the deviation of theoretical solutions from the reality.

To assess the practicality and input orientation of the results solved by coalitional game, the paper designed and conducted an experiment to simulate the decision making process of players aiming to form a grand coalition. The experiment emphasizes on the expectations of players regarding their input and output to the coalition. The negotiation process was not conducted because we aimed to propose a solution fairly expressed by the players’ profit expectations free from other objective influence factors such as negotiation skills and status inequality. But in reality these factors exist and influence the profit allocation scheme in every area. The understanding of fairness varies from different perspective and in different scenarios.

The experiment brought in the resource devotion representing the inputs to the alliance. The quantification methods will be specified in further research. The evaluation and influence assessment on other important factors regarding the profit allocation will be investigated as well.

REFERENCES


