An Image Enhancement Method Based on Improved Fuzzy Set

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Abstract
Affected by conditions of scenes, the images shot in many high dynamic ranges, dark environments or special lights are not good in visual effects and they will require enhancement processing to adjust the dynamic range or extract a uniform color sense to meet the requirements. Image enhancement refers to the image processing method to highlight certain image information and weaken or remove irrelevant information or convert the original image into a form that is more suitable for analytical processing by humans or machines with specific methods according to specific requirements. As the image is fuzzy in its essence, it has fuzziness in defining such image features as boundary, regions and textures and in the explanation of processing result of image bottom. Therefore, this paper proposes an image enhancement method based on improved fuzzy set. This method partitions the image into sub-images based on the fuzzy membership function rules and performs image enhancement on the sub-images so as to achieve effective image sharpening. According to the grey-level characteristics of the image, this paper cites the relative grey level of pixel as the fuzzy feature and performs fuzzy enhancement with improved membership function in order to avoid loss of much grey-level information after enhancement. While achieving optimization in fuzzy membership function criteria, it preserves as many image edge locations and detail information as possible; meanwhile, it confirms the selection of the optimal parameters, ensures the quality of enhanced image and improves the feasibility and efficiency of the algorithm. It can be seen from the experiment simulation result that whether from subjective analysis or objective judgment, the experiment result shows that the method of this paper is effective and feasible for image enhancement.

Key words: Image Enhancement, Fuzzy Set, Membership Function.

1. INTRODUCTION

In image processing, image enhancement is one of the techniques most frequently used in image pre-processing and image enhancement technology plays an important role in improving image quality. Image enhancement adjusts or sharpens certain image features such as edge, outline and contrast to facilitate display, observation or further analysis and processing(Takatsu and Ueyama et al., 2016). Image enhancement will not increase relevant information in image data, but it will increase the dynamic ranges of the selected features so as to make these features easier to be detected or recognized. The main purpose of image enhancement is to apply a series of technical means to improve the information carried in the image, eliminate the useless information of the image, remove noises, restore useful information, suppress unnecessary distortion or enhance the important image features for subsequent processing and convert the image into a form more suitable for analytical processing of humans or computers. So far, image enhancement still has many questions to be solved. The theory of image enhancement is to be improved. And there are still many things to be further studied. Fuzzy set theory analyzes and processes fuzzy things with mathematical methods. On one hand, it adopts the basic ideas and methods of classic mathematics and on the other hand, it doesn’t define fuzzy subset in the way of classic mathematics. In fuzzy mathematics, membership is built upon fuzzy set and membership function is the key to describe fuzziness. In recent years, fuzzy set theory can be successfully applied in the field of image processing and shows the processing better than conventional methods(Corbin and Reyna et al., 2015). The root reason is that the uncertainty of the image is usually caused by fuzziness. The fuzzy method of image enhancement is to achieve by modifying the pixel value in the fuzzy feature field.

The frequently used image enhancement methods include grey-level transformation, histogram modification, image sharpening, noise removal, geometric distortion correction, frequency-domain filtering and color enhancement. As image enhancement is closely related to the object characteristics of interest, the habits of observers and processing intent, although the processing methods may be diversified, it is strongly targeted. So, the applications of image enhancement algorithms are also targeted and there is no universal enhancement algorithm suitable for various applications(Wang and Jia et al, 2015). In order to improve the image quality with
different specific purposes, many image enhancement algorithms have been brought into being. The conventional image enhancement methods basically can be divided into two kinds: spatial-domain image enhancement and frequency-domain image enhancement. Spatial domain refers to the space constituted by pixels and spatial-domain methods directly operate on the pixel points of the image. Spatial-domain image enhancement directly performs operations on grey level of image pixels, e.g. grey-level transformation, histogram equalization, spatial smoothing and sharpening of image as well as pseudo-color processing. Frequency-domain image enhancement usually transforms the image into frequency domain with the help of such transformation methods as Fourier analysis before filtering and other processing, including low-pass filtering, high-pass filtering and homomorphic filtering (Ghani and Isa, 2014). Currently, the hot research includes the enhancement algorithm based on local histogram equalization, the enhancement method based on wavelet transformation and the enhancement method based on fuzzy mathematics. In 1965, Professor L.A.Zadeh from University of California-Berkeley, USA, published the first paper about fuzzy theory and proposed the concept of fuzzy set to represent fuzzy things for the first time from the perspective of fuzzy theory in order to make up for the defect to use binary logic to describe things. Ever since Zadeh came up with fuzzy set and convex fuzzy set, Biswas introduced the idea of anti-fuzzy subset for the first time in 1990, based on which, the study on anti-convex fuzzy set based on convex fuzzy set has drawn increasing attention of many scholars (Brainerd and Reyna, 2015). In 1996, an Indian named Sarkar Debranjan combined convex fuzzy set and concave fuzzy set, provided the definition of concave-convex fuzzy set and conducted the research (Pourrahmani and Delavar et al., 2015). To improve the traditional fuzzy enhancement methods requires identifying the changes of parameters by comparing the enhanced image. Further study is required on how to achieve the selection of the optimal parameters for membership function.

This paper first introduces fuzzy logic and fuzzy set and the principle of maximum membership. Then it states image grey-level transformation and enhancement theory and summarizes the existing image enhancement methods and their characteristics. On this basis, it proposes an image enhancement method based on fuzzy set, including how to perform fuzzy feature extraction on the image, numerical value modification of membership function and inverse transformation of fuzzy field. Finally, it analyzes the experiment result, demonstrates the accuracy and viability of the algorithm of this paper and better preserves image edges and details.

2. FUZZY LOGIC AND PRINCIPLE OF MAXIMUM MEMBERSHIP

Fuzzy logic is the approach and tool to analyze uncertain and imprecise information by simulating the way of thinking of humans. In essence, fuzzy logic is nothing but fuzzy. In fact, it is not a “fuzzy” logic, but a logic that is used to manipulate fuzzy phenomena or events so as to eliminate fuzziness. In fuzzy logic, membership function is used to reflect the degree to which the elements in fuzzy set belong to the set.

2.1. Fuzzy Logic and Convex Fuzzy Set

Based on the description of natural language, fuzzy logic can be built on the foundation of experts’ experience, accept the use of imprecise data and can build modeling on any complex non-linear functions. Defuzzification is to make the decision of defuzzification on the output of fuzzy reasoning; that it to say, in a range of output, it find the most representative and accurate output value. Fuzzy system can describe and manipulate the problems in a fast and convenient manner because of the following reason.

If \( A \) is a fuzzy subset with real number field \( R' \) as the fuzzy subset of the domain of discourse, the membership function is \( \mu_A(x) \), and for \( \forall \ a < x < b \), \( \mu_A(x) \geq \min(\mu_A(b), \mu_A(a)) \), then \( A \) is a convex fuzzy set.

The following Fig.1 shows convex fuzzy set and non-convex fuzzy set.

![Convex fuzzy set and non-convex fuzzy set](image.png)

**Figure 1.** Convex fuzzy set and non-convex fuzzy set
\( \lambda \) cut set of convex fuzzy set will be an interval, on the contrary, the fuzzy set with \( \lambda \) cut set as an interval will be convex fuzzy set. If \( A \) and \( B \) are convex fuzzy sets, then \( A \cap B \) will also be a fuzzy set. If they are non-convex fuzzy sets, it will be a non-convex fuzzy set. Fuzziness reflects the uncertainty of events, but such uncertainty is different from randomness. Randomness reflects the objectively natural uncertainty or the contingency of time while fuzziness reflects the uncertainty of people’s subjective understanding, namely the uncertainty in people’s understanding of linguistic meaning on relevant definition of time or description of concepts (Qin and Liu et al., 2015).

2.2. Fuzzy Extremum of Bounded Function

In the process of fuzzy reasoning, conduct the reasoning according to various control rules, combine the results together and produce a set of “fuzzy reasoning output”. Fuzzy logic reasoning is one of uncertainty reasoning methods based on fuzzy logic. This method, on the premise of fuzzy judgment, operates fuzzy linguistic rules and reasons to a new and approximate fuzzy judgment conclusion. To decide whether fuzzy logic reasoning is conducted doesn’t mean whether the concept of fuzziness is adopted in the premise and the conclusion, but whether the reasoning process is fuzzy; to be specific, whether the reasoning rules are fuzzy.

Assume \( f : X \rightarrow R \) (\( R \) is a real number set) and \( x \rightarrow y = f(x) \) is a bounded function, to seek the normal extremum of function \( f(x) \) is to find the \( x' \) that meets \( f(x') = \max\{f(x) \mid x \in X\} \).

The \( x' \) that meets the above formula is the maximum point of \( f(x) \) in \( X \). \( f(x') \) is the maximum value, but there may be more than one maximum point.

Assume that the set of all maximum points of \( f(x) \) is \( M_f = \{x' \mid f(x') = \max\{f(x) \mid x \in X\}\} \), \( M_f \) is called as the superiority set of \( f(x) \). When \( x \in M_f \), the function reaches the maximum value \( f(x) \) in \( x \) and \( x \) makes \( f(x) \) reach the optimum. When \( x \notin M_f \), though \( f(x) \) is not the maximum value, for different \( x \), the differences between \( f(x) \) and the maximum value can be different. In other words, for the \( x \) that don’t belong to \( M_f \), their “superiority” may be different. In order to reflect different superiority of different points of \( X \), fuzzify the superiority set \( M_f \), with which fuzzify the extremum (Sevastjanov and Dymova, 2015).

2.3. Principle of Maximum Membership

Fuzzy set makes a certain element can belong to a certain set to a certain degree and the degree that a certain element belongs to a certain set is depicted or described by the membership, a numerical value between “0” and “1”. To map a specific element to a proper membership is achieved by membership function. If the standard types are some fuzzy sets to represent fuzziness and when the object to be recognized is a certain element (individual) of the domain of discourse, the membership is not 1 usually because the object to be recognized doesn’t belong to a certain standard type absolutely. People usually use what is referred to as “the principle of maximum membership” to identify such kind of problems. This kind of method (including the “principle of threshold” below) manipulates individual identification and it is called direct method. Membership function can be a curve in any shape, which is determined by whether the shape makes it simple, convenient, fast and effective for us to use, and its only constraint condition is that the range of value of the membership function is \([0,1]\). The frequently used membership functions in fuzzy system include Gaussian membership function, triangle-shape membership function and trapezoid-shape membership function. The principle of maximum membership is classified as follows.

Assume that \( A_1, A_2, \ldots, A_n \in F(U) \) is \( n \) standard types and \( x_0 \in U \), if \( A_i(x_0) = \max\{A_i(x_0) \mid 1 \leq k \leq n\} \), then it is considered that \( x_0 \) is a relative membership of the type represented by \( A_i \). Through fuzzification, the exact value can be converted into a fuzzy subset in the standard domain of discourse. After the exact value is converted to a basic element in the standard domain of discourse through correspondence, the fuzzy subset that has the maximum membership in this element is the corresponding fuzzy subset of that exact value. In short, fuzzification is to arrange the input/output variables into different memberships through various classifications (Tang, 2015).

3. IMAGE TRANSFORMATION AND ENHANCEMENT

The common grey-level transformation is to directly modify the input/output mapping relation of grey level, enhance the image regions of interest and suppress the regions of no interest so as to enhance the contrast of the image. Proportional linear transformation is to manipulate the pixels one by one in every linear section and it
can extend the dynamic range of grey-level value of original image to the designated scope or the entire dynamic range (Celik, 2014).

Assume that two grey-level intervals are given, the grey-level scope of the original image \( f(x, y) \) is \([a, b]\) and if the grey level of the transformed image \( g(x, y) \) is expected to be extended to \([c, d]\), the linear transformation indicated in the following formula can be obtained according to the linear equation.

\[
g(x, y) = \frac{d - c}{b - a} [f(x, y) - a] + c
\]  

(1)

That is to say, a certain interval \([a, b]\) of brightness value of input image can be extended to the interval \([c, d]\) of the output value. To use proportional linear grey-level transformation to make a linear extension to the grey level of every pixel will effectively improve the visual effects of the image (Sundaram and Ramar et al., 2011).

If the grey level of the image is within the range of \(0 \sim M\), the grey levels of most pixels are distributed within the interval of \([a, b]\) with a very small portion beyond this interval. In order to improve the enhancement, the mapping relation is as follows.

\[
g(x, y) = \begin{cases} 
  c & 0 \leq f(x, y) \leq a \\
  \frac{d - c}{b - a} [f(x, y) - a] + c & a < f(x, y) < b \\
  d & b < f(x, y) \leq M 
\end{cases}
\]  

(2)

To analyze some special circumstances, make \( k_1 = c / a \), \( k_2 = (d - c) / (b - a) \), \( k_3 = (\max g - d) / (\max f - b) \), in other words, they correspond to the slopes of straight lines respectively.

When \( k_1 = k_2 = 0 \), as shown in Fig.2(a), it means that it has no interest to the grey level of the original image beyond \([a, b]\) and make them 0 while for the grey level of the original image within the range of \([a, b]\), convert into the grey level in the new image uniformly.

When \( k_1 = k_2 = k_3 = 0 \), but \( c = d \), as suggested in Fig.2(b), it means that it only has interest in the grey level within the range of \([a, b]\) and that they are all white while the rest black, at this time, the image will become a binary image, this operation is called as grey-level (or window) slice.

When \( k_1 = k_3 = 1 \) and \( c = d = \max g \), as indicated in Fig.2(c), it means that while preserving the background, upgrade the grey level of the pixels within the range of \([a, b]\). It is also a kind of window or grey-level slice (Nam-Thai and Moon et al., 2012).

**Figure 2.** Three linear transformations

**4. IMAGE ENHANCEMENT METHODS BASED ON FUZZY SET AND EXPERIMENT SIMULATION TEST**

**4.1. Image Enhancement Methods Based on Fuzzy Set**

The membership function of fuzzy set used in this paper is \( f : X \rightarrow R \) and it is a bounded function. The membership function \( M_f \) is defined as follows.
\[
M_f(x) = \frac{f(x) - \min \{ f(x) \mid x \in X \}}{\max \{ f(x) \mid x \in X \} - \min \{ f(x) \mid x \in X \}} \quad (\forall x \in X)
\]

\(M_f\) is the unconditioned fuzzy superiority set of \(f\) and \(f(M_f)\) is the unconditioned fuzzy maximum of \(f\). Here, \(f(M_f) \in F(R)\) and its membership function, according to the extension principle, is as follows.

\[
f(M_f)(y) = \vee \{ M_f(x) \mid f(x) = y \} \quad (\forall \varphi = 0)
\]

When \(x = x_1\) is the maximum point of \(f(x)\), namely when \(f(x_1) = \max \{ f(x) \mid x \in X \} \), \(M_f(x_1) = 1\) and when \(x = x_2\) is the minimum value of \(f(x)\), namely when \(f(x_2) = \min \{ f(x) \mid x \in X \} \), \(M_f(x_2) = 0\) and the necessary and sufficient condition of \(f(x_1) \leq f(x_2)\) is as follows.

\[
M_f(x_1) \leq M_f(x_2) \quad (\forall x_1, x_2 \in X)
\]

The image enhancement based on fuzzy set mainly includes the following three steps: extraction of fuzzy features, modification of membership function value and inverse transformation of fuzzy field.

1. **Extraction of fuzzy features of image.**
   Transform the image from spatial domain to fuzzy field through the following transformation.

   \[
   \mu_{mn} = G(g_{mn}) = \left[ 1 + \frac{s_{mn} - g_{mn}}{F_d} \right]^\gamma
   \]

   In this formula, \(F_e\) and \(F_d\) are the transformation coefficients, \(s_{mn}\) is the maximum grey-level value of the image and \(g_{mn}\) is the grey-level value of the current pixel.

2. **Modification of membership function value.**
   Through the transformation below, revise the membership with the regression calling of fuzzy enhancement operator.

   \[
   T(\mu_{mn}) = \begin{cases} 
   24[\mu_{mn}]^3 & 0 \leq \mu_{mn} \leq 0.5 \\
   1 - 24[1 - \mu_{mn}]^3 & 0.5 < \mu_{mn} \leq 1 
   \end{cases}
   \]

   The key to fuzzy enhancement is to increase the membership bigger than 0.5 and reduce the membership smaller than 0.5 with fuzzy enhancement operator.

3. **Inverse transformation of fuzzy field.**
   Produce new grey levels through inverse transformation so as to transform the data from fuzzy field to the spatial domain of the image.

   \[
g'_{mn} = G^{-1}(\mu'_{mn}) = g_{mn} - F_d[(\mu'_{mn})^{\frac{1}{\gamma}} - 1]
   \]

   The grey-level enhancement of the image is to enhance the contrast of brightness and darkness of the image in order to strengthen the different object features in brightness and darkness. As the description of grey levels of an image is limited, the core idea of grey-level enhancement of image is to enhance the important object information by suppressing the unimportant information.

**4.2. Experiment Simulation Test**

The test experiment uses Matlab software and chooses Office and Canoe as test images for processing with the method of this paper. Fig.3 and Fig.4 are the enhancement result and RGB histogram.
Figure 3. Enhancement result of Office image

Figure 4. Enhancement result of Canoe image
It can be found from the experiment results that for different types of images, the method of this paper can better adjust the weight vector of membership function, obtain the optimal value for the parameters and get the better color image enhancement. As for the over-dark phenomena in the object region, the loss of some detailed information of the object and blocking effect, it is shown by the processing result of the algorithm of this paper that the brightness is distributed in a coordinated manner as a whole in the image, such detailed information as edge and outline enhanced significantly and both the object and background information better recovered, suggesting that the algorithm can effectively enhance the image contrast so as to improve the visual effect of the image.

5. CONCLUSIONS

The image processing technique based on fuzzy set is a research direction worthy of attention. The application of fuzzy set can usually obtain better result than conventional methods and it can solve the incompleteness, uncertainty and difficulty in modeling. In recent years, some research on the image processing based on fuzzy set has made some achievements, but more in-depth study and exploration are required in image enhancement. This paper has proposed an image enhancement method based on improved fuzzy set and it can further enhance the quality and visual effect of the image after histogram equalization. The simulation experiment has proven that the algorithm of this paper is correct and it results in excellent image enhancement when used in enhancing various images.

REFERENCES