doi:10.21311/002.31.11.13

A PML algorithm for positron emission tomography based on Poisson-modified total variation model

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Abstract

Recently, positron emission tomography (PET) has been widely used in medical image reconstruction. However, because of low tracer dosages and other reasons, the PET images are usually strongly polluted by noise, especially Poisson noise. The results of clinical diagnosis will be seriously affected by this noise.In order to suppress Poisson noise in reconstructed images, a new penalized maximum likelihood algorithm is proposed in this paper. It combines the Poisson-modified total variation model with the maximum likelihood expectation-maximization (MLEM) algorithm. Iterations of the proposed method can be divided into two steps: firstly, reconstructing image with the MLEM algorithm; secondly, suppressing Poison noise with the Poisson-modified total variation model. Experimental results demonstrate that the proposed method can effectively suppresses Poison noise in the PET images and is superior to many existing excellent algorithms.

Keywords: Image reconstruction, Positron emission tomography, Penalized maximum likelihood, Poisson-modified total variation, Poison noise.

1.INTRODUCTION

As one of the new imaging technologies that has been applied in clinical practice since computed tomography (CT) and magnetic resonance imaging (MRI) were implemented, positron emission tomography (PET) has been widely used in tumour cell detection, disease diagnosis in the heart and nervous system and psychiatric diseases, as well as in new drug development.

The purpose of PET imaging is to obtain an intracorporal distribution produced by a radioactive substance. Therefore, how to reconstruct a high quality image based on the scan data has always been an important research subject in the field of PET. However, the presence of noise(especially Poisson noise) on PET images is unavoidable. It may be introduced during the image formation process, low tracer dosages, and so on. This noise will seriously affect the results of clinical diagnosis because even a small amount of noise is harmful when high accuracy is required.

The maximum-likelihood expectation-maximization (MLEM) algorithm is a classic method in PET imaging, which seeks an image that makes the measured data most likely to occur when the measured data follows Poisson distribution(Shepp and Vardi, 1982). One defect of this algorithm is that the quality of the reconstructed image is very poor and unstable when the scan data is seriously polluted by noise(Tang and Chen, 2016). In essence, the MLEM algorithm is an ill-posed problem. Today, anill-posed image reconstruction problem, such as MLEM, can be transformed into a well-posed one through the use of a regularization term. That is referred to as a penalized maximum likelihood(PML) or Bayesian algorithm(Green, 1990; Shen et al., 2016). The PML algorithm has been widely studied because of its effective noise reduction ability and unique solution(Wang et al.,2015;Fathi et al., 2015). In 1990, Green first proposed a Bayesian algorithm, known as the one-step-late (OSL) algorithm (Guiet al.,2012). The key of this algorithm is to find an appropriate energy function, which determines the performance of the algorithm. Subsequently, Alenius used a median filter instead of the derivative of the energy function in the OSL algorithm and proposed median root prior (MRP) algorithm(Alenius and Ruotsalainen, 1997).However, the images reconstructed by MRP are still noisy because the median filter cannot effectively remove Gaussian and Poisson noise in PET images. Recently, in order to produce high quality PET images, image reconstruction based on an AD filter has become the research focus(Dong et al., 2015; Guiet al.,2012;Yan and Yu, 2007;He and Huang, 2015). In 2007, Yan and Yu proposed a PDE median algorithm that combined an AD filter with MRP and could obtain acceptable reconstruction results if the parameters were set appropriately(Yan and Yu, 2007). Later, He and Huang proposed the MRPAMD algorithm on the basis of PDE median (He and Huang, 2015). Compared to the PDE median algorithm, the MRPAMD method is less sensitive to the value of the gradient threshold and the adjustment of the diffusion number.

The noise within the PET images is mainly Poisson noise, while some traditional PET image reconstruction algorithms, such as MLEM, OSL, and MRP, have better suppression effects on the general additive noise, but not on Poisson noise. Therefore, how to suppress the Poison noise in PET images is an important research topic (Teymurazyan et al., 2013;Wei and Liu, 2014; Singer and Wu, 2013).

In this paper, we proposed a new PML algorithm for PET image reconstruction by combining the Poisson-modified total variation model (Leet al.,2007) with the MLEM algorithm. The proposed method can effectively suppress Poison noise and improve the quality of PET images. In Section 2, the MLEM algorithm is proposed. The Poisson-modified total variation model is introduced in Section 3. In Section 4, the numerical solution of total variation model is described. In Section 5, our proposed algorithm is proposed. Simulation experiment results are given in Section 5. Finally, Section 6 is the conclusion.

2. MLEM ALGORITHM

In PET image reconstruction, the number of photons captured by the radial bin i is follows Poisson distribution (Shepp and Vardi, 1982):

$$g_i \sim \text{Poisson}(\sum_{j=1}^{M} H_{ij} f_j) \tag{1}$$

where H_{ij} is the probability of photons emitted by pixel j, which can be detected by the radial b in i, and g_i is the number of photons captured by the radial bin$i$. The likelihood function is given by

$$L(f) = p(g \mid f) = \prod_{i=1}^{N} \exp\left(-\sum_{j=1}^{M} H_{ij} f_j\right) \frac{\left(\sum_{j=1}^{M} H_{ij} f_j\right)^{g_i}}{g_i!} \tag{2}$$

Where f and g denote emission image and measured data, respectively. Equation (2) is the well-known maximum likelihood (ML) algorithm. In order to solve the Equation (2), Shepp and Vardi have proposed the expectation maximization (EM) algorithm. The discrete form of the MLEM is the following equation:

$$f_j^{k+1} = f_j^k \frac{\sum_{i=1}^{N} H_{ij} \frac{g_i}{\sum_{l=1}^{M} H_{il} f_l^k}}{\sum_{i=1}^{N} H_{ij}} \tag{3}$$

3.POISSON-MODIFIED TOTAL VARIATION MODEL

In 1992, Rudin, et al., (1992) proposed the total variation(TV) model by modelling images in BV space.In the TV model, *f* is the noise image, *u* is the original image, and *n* is the additive noise, so the noise image can be expressed follows:

$$f = u + n \tag{4}$$

The TV model can be simply expressed as the following optimization problem with noise constraints:

$$\begin{aligned} &\min_u \int_\Omega \sqrt{u_x^2 + u_y^2}\,\mathrm{d}x\mathrm{d}y \\ &s.t. \int_\Omega u\,\mathrm{d}x\mathrm{d}y = \int_\Omega f\,\mathrm{d}x\mathrm{d}y, \frac{1}{|\Omega|}\int_\Omega (f-u)\,\mathrm{d}x\mathrm{d}y = \sigma^2 \end{aligned} \tag{5}$$

where,$\Omega$is the image domain and $|\Omega|$is the area of the image.

According to the total variation of the translational invariance, the TV model can be equivalent to the following unconstrained optimization problems:

$$F(u) = \min_u \int_\Omega \sqrt{u_x^2 + u_y^2} + \frac{\lambda}{2}(u-f)^2\,\mathrm{d}x\mathrm{d}y \tag{6}$$

where, $\int_\Omega \sqrt{u_x^2 + u_y^2}\,\mathrm{d}x\mathrm{d}y$ is a regularization term, $\int_\Omega \frac{\lambda}{2}(u-f)^2\,\mathrm{d}x\mathrm{d}y$ is a data-fidelity term, and λ is a regularization parameter that determines the relative importance of the two terms. Theoretically, the most ideal method of λ is to take a larger value at the edges and a smaller value in the flat areas in order to achieve the purpose of suppressing noise and protecting the edges of the image.In the TV model, a dynamic method of λ is given:

$$\lambda = -\frac{1}{2\sigma^2}\int\left[\sqrt{u_x^2+u_y^2} - \left(\frac{(u_0)_x u_x}{\sqrt{u_x^2+u_y^2}} + \frac{(u_0)_y u_y}{\sqrt{u_x^2+u_y^2}}\right)\right]\mathrm{d}x\mathrm{d}y \tag{7}$$

Like many other image denoising models, the TV model has a positive effect on the independent signal additive noise. However, some of the images (such as radioactive images) mainly contain noise that is signal dependent, such as Poisson noise, etc. In order to solve this problem, TRIET LE proposed the Poisson-modified total variation (PMTV) algorithm by modifying the data fidelity term in the TV model. The PMTV algorithm can be simply expressed as the following optimization problem:

$$\min_u \int_\Omega (u - f \lg u) \mathrm{d}x\mathrm{d}y + \beta \int_\Omega |\nabla u| \mathrm{d}x\mathrm{d}y \quad (8)$$

The corresponding Euler Lagrange equation is as follows:

$$0 = \mathrm{div}(\frac{\nabla u}{|\nabla u|}) + \frac{1}{\beta u}(f - u) \quad (9)$$

Notice that, compared to the PMTV algorithm and the TV model, only the regularization parameters are different. In the PMTV algorithm, the regularization parameter$\lambda=1/\beta u$, depends on the reconstructed image u. Because of the existence of parameterλ, the PMTV model has better a denoising effect for Poisson noise.

4. NUMERICAL SOLUTION OF THE TV MODEL

Based on the expression of the TV model, it is known that the TV model is nonlinear and non-differentiable. Therefore, it is very difficult to solve the TV model (Yakusak et al., 2015; Liang, 2015; Vidhya vathi et al. 2015).Next, we introduce the method of solving the TV model.

In order to solve the TV model, the Euler Lagrange equation of the Formula (6) is obtained first. Assuming $w \in C_0^1(\Omega)$, then

$$\begin{aligned} g(\varepsilon) &= F(u + \varepsilon w) \\ &= \int_\Omega \sqrt{(u_x + \varepsilon w_x)^2 + (u_y + \varepsilon w_y)^2} \mathrm{d}\Omega + \frac{\lambda}{2} \int_\Omega (f - u - \varepsilon w)^2 \mathrm{d}\Omega \end{aligned} \quad (10)$$

Assuming $g'(0)=0$, then

$$\int_\Omega \left[-\mathrm{div}(\frac{\nabla u}{|\nabla u|}) - \lambda(f - u) \right] \cdot w \mathrm{d}\Omega = 0 \quad (11)$$

According to the arbitrary nature of w

$$-\mathrm{div}\left(\frac{\nabla u}{|\nabla u|}\right) - \lambda(f - u) = 0 \quad (12)$$

The above formula is the Euler Lagrange equation of the TV model.

By introducing time variable t and using the gradient descent method, Formula (12) can be converted to the following:

$$\frac{\partial u}{\partial t} = div\left(\frac{\nabla u}{|\nabla u|}\right) + \lambda(f - u) \quad (13)$$

In order to avoid the singularity caused by $|\nabla u|=0$,introduce a parameter$\xi>0$, then

$$\frac{\partial u}{\partial t}=\mathrm{div}\left(\frac{\nabla u}{\sqrt{u_x^2+u_y^2+\xi}}\right)+\lambda(f-u) \tag{14}$$

In practical application, we need to make a discretization of the above formula. The specific process is as follows:

Assume that the size of the image u is $N\times N$ pixels, set up $(\nabla u)_{i,j}=(((\nabla u)^1_{i,j}), ((\nabla u)^2_{i,j}))$, and

$$(\nabla u)^1_{i,j}=\begin{cases} u_{i+1,j}-u_{i,j}, & i<N; \\ 0, & i=N. \end{cases}$$
$$(\nabla u)^2_{i,j}=\begin{cases} u_{i,j+1}-u_{i,j}, & j<N; \\ 0, & j=N. \end{cases} \tag{15}$$

Then Formula (14) can be transformed into

$$\frac{u_{n+1}-u_n}{\Delta t}=\mathrm{div}\left(\frac{(\nabla u_n^1,\nabla u_n^2)}{\sqrt{u_x^2+u_y^2+\xi}}\right)+\lambda(f-u) \tag{16}$$

Set up

$$p=\frac{(\nabla u_n^1,\nabla u_n^2)}{\sqrt{u_x^2+u_y^2+\xi}},$$
$$p^1=\frac{\nabla u_n^1}{\sqrt{u_x^2+u_y^2+\xi}},$$
$$p^2=\frac{\nabla u_n^2}{\sqrt{u_x^2+u_y^2+\xi}}. \tag{17}$$

Then

$$\frac{u_{n+1}-u_n}{\Delta t}=\mathrm{div}(p)+\lambda(f-u)=\mathrm{div}((p^1,p^2))+\lambda(f-u) \tag{18}$$

where

$$(div(p))_{i,j}=\begin{cases} p^1_{i,j}-p^1_{i-1,j}, 1<i<N; \\ p^1_{i,j}, \quad i=1; \\ -p^1_{i-1,j}, \quad i=N. \end{cases}+\begin{cases} p^2_{i,j}-p^2_{i,j-1}, 1<j<N; \\ p^2_{i,j}, \quad j=1; \\ -p^2_{i,j-1}, \quad j=N. \end{cases} \tag{19}$$

In summary, the discrete formula of the TV model can be expressed as follows:

$$u_{n+1} = u_n + \Delta t \left[\operatorname{div}\left(\frac{\nabla u}{\sqrt{u_x^2 + u_y^2 + \xi}} \right) + \lambda(f - u) \right] \tag{20}$$

5. PROPOSED ALGORITHM

In the process of reconstructing the image, the MLEM algorithm takes into account the statistical characteristics of the noise and improves the quality of the reconstructed image to a certain extent. However, this algorithm is not ideal for Poisson noise, and we need to do more research.

In this paper, a new PML algorithm was proposed by combining the Poisson-modified total variation model with the MLEM algorithm. The proposed algorithm is called the MLEM-PMTV algorithm. Iterations of the new method can be divided into two steps: firstly, reconstruct the image with the MLEM algorithm, and secondly, suppress Poison noise with the Poisson-modified total variation model. The specific iterative equations are as follows:

$$f_j^{k+1,l} = f_j^{k,l} \frac{\sum_{i=1}^{N} H_{ij} \frac{g_i}{\sum_{l=1}^{M} H_{il} f_l^{k,l}}}{\sum_{i=1}^{N} H_{ij}}, \tag{20}$$

$$u_j^{k+1,l} = f_j^{k+1,l} + \Delta t * \operatorname{div}\left(\frac{\nabla f_j^{k+1,l}}{\sqrt{\left|\nabla f_j^{k+1,l}\right| + \xi}} \right), \tag{21}$$

$$f_j^{k+1,l+1} = u_j^{k+1} + \Delta t * \lambda(f_j^{k+1,l} - u_j^{k+1}). \tag{22}$$

where, k and l denote the number of iterations of the MLEM algorithm and PMTV model. Notice that, in a real image reconstruction process, the original noise image f is unknown, so we can only use the estimated method in order to estimate the value of the f. In this algorithm, assume $f = f^{k+1,l}$.

6. SIMULATION EXPERIMENTS

In the simulation experiments, we first used a test image that was a computer-generated modified Shepp-Logan phantom, which is shown in Figure 1. The size of the image phantom is 128×128 pixels. Assuming that the size of the projection parameter is128×128, that is, there are 128 projection directions (evenly distributed between0~π), and each projection direction has 128 radial bins. Using the formula $g=Hf$ in order to generate the observation data without noise,the Poisson noise was added into the projection data. In the simulation experiments, the total amount of photons collected by radial bins was about6×10^5 pairs.

Figure 1. Modified Shepp-Logan phantom.

In order to examine the validity of the proposed algorithm, we compared reconstructed images produced by the proposed algorithm with those produced by different algorithms, such as MLEM-TV, MRP, and MLEM. In order to guarantee the fairness of the experiments, we stipulated that the iterations of all algorithms were set to 50 times. In the MLEM-PMTV and MLEM-TV algorithms, the number of iterations of the PMTV and TV denoising models were set to I=40,Δtwere set to 0.8, and the regularization parameters are set to 0.3 and 0.04, respectively (Leet al.,2007). In the MRP algorithm, the transcendental parameter was set to 0.1.

The modified Shepp-Logan phantom, as reconstructed by four algorithms, is shown in Figure 2, and Figure 3 is the zoomed-in images of Figure 2. These figures show that the quality of the image reconstructed by the MLEM is the worst. It contains a lot of noise and edge blur. The MLEM-TV and MRP algorithms performed better than MLEM, but the images reconstructed by these two algorithms still contained significant noise. In contrast, the proposed algorithm resulted in the best image quality; the noise in the image is relatively small, and the edge is clear. Overall, the image reconstructed by the MLEM-MPVT algorithm is the best from the perspective of subjective visual effects.

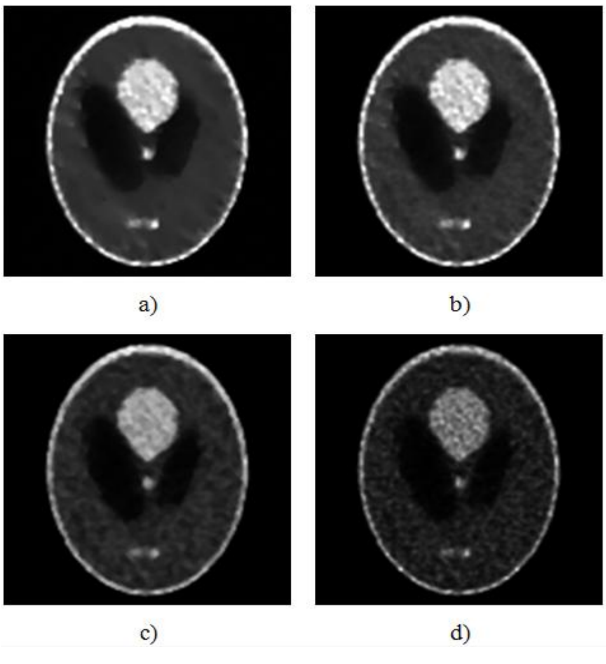

Figure 2. The modified Shepp-Logan phantom reconstructed by four algorithms: a) MLEM-PMTV; b) MLEM-TV; c) MRP; d) MLEM.

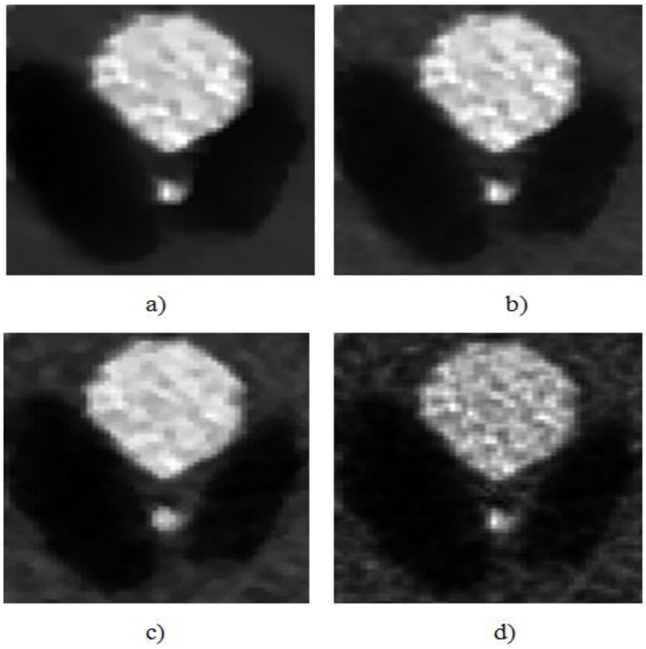

Figure 3. The zoomed-in images of Figure2: a) MLEM-PMTV; b) MLEM-TV; c) MRP; d) MLEM.

Next, we analysed the effectiveness of the algorithm by calculating the NRMSR and SNR values of the reconstructed images. Figure 4 shows the plots of NRMSE along with iterations of the four algorithms. From this figure, we can see that the NRMSE value of the new algorithm is the smallest. This shows that the image reconstructed by the proposed algorithm is closest to the original image. Similar conclusions can be obtained by analysing the SNR curve in Figure 5.

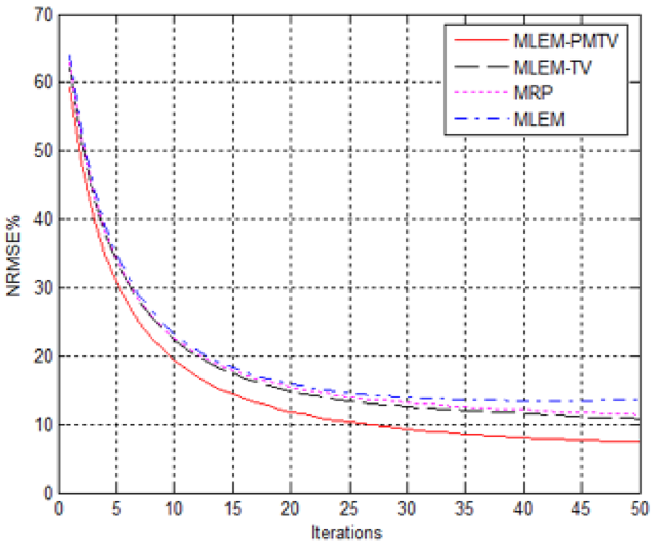

Figure 4. The plots of NRMSE along with iterations for modified Shepp-Logan phantoms reconstructed by four algorithms.

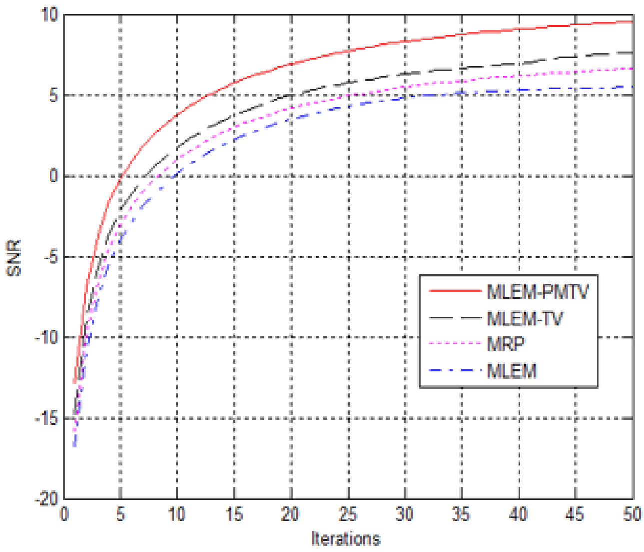

Figure 5. The plots of SNR along with iterations for modified Shepp-Logan phantoms reconstructed by four algorithms.

In order to further illustrate the effectiveness of the proposed algorithm, we use the thoraxphantom as a test model. The size of the thoraxphantom is 128×128 pixels, and which is shown in Figure 6.The method of obtaining the projection data is similar to that of the previous method. The total amount of photons collected by radial bins was about 5.2×10^5 pairs.

Figure 6. Thorax phantom.

We also compared the new algorithm to the MLEM-TV, MRP, and MLEM algorithms. In the simulation experiments, the iteration numbers of all the algorithms were set to 50 times. For the MLEM-PMTV and MLEM-TV algorithms, the number of iterations of the PMTV and TV denoising models were set to $l=40$,Δt were set to 0.7, and the regularization parameters were set to 0.3 and 0.04, respectively. For the MRP algorithm, the transcendental parameter was set to 0.1. The thorax phantoms reconstructed by these four algorithms are shown in Figure 7, and Figure 8 shows the zoomed-in images of Figure 7. These figures show that the image reconstructed by the proposed algorithm has the least noise and its overall visual effect is better than those of the other three algorithms.

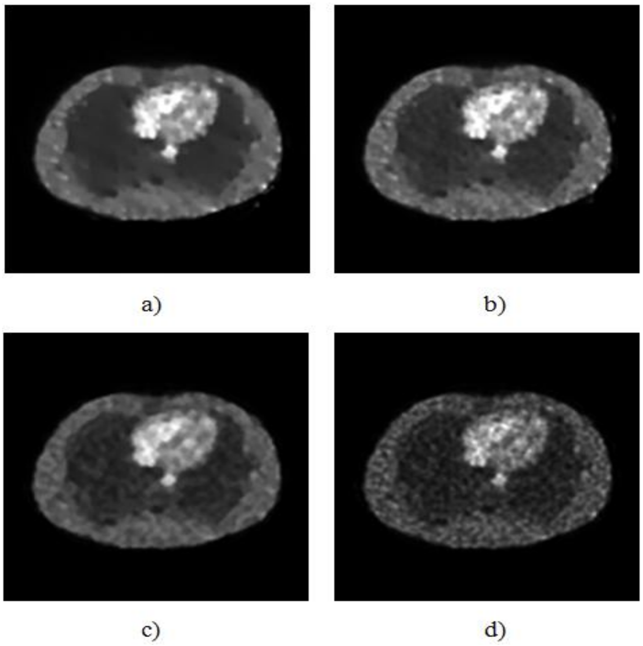

Figure 7. The thorax phantom reconstructed by four algorithms: a) MLEM-PMTV; b) MLEM-TV; c) MRP; d) MLEM.

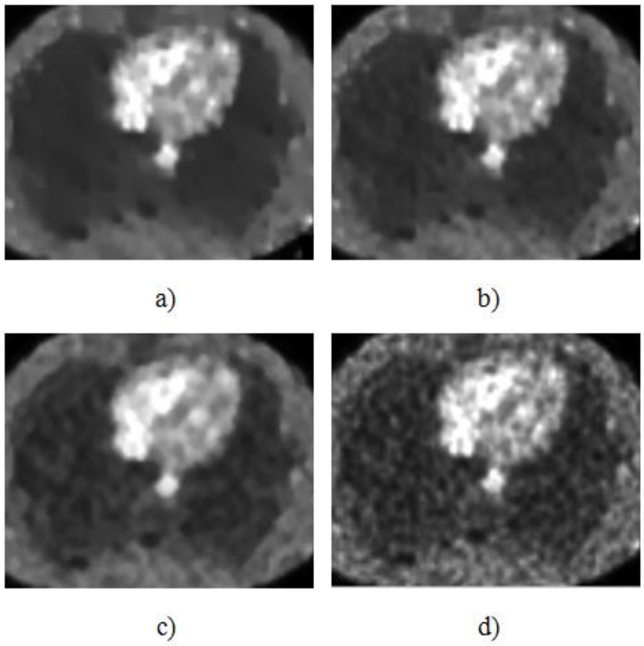

Figure 8. The zoomed-in images of Figure7: a) MLEM-PMTV; b) MLEM-TV; c) MRP; d) MLEM.

Figure 9 and Figure 10 are the plots of NRMSE and SNR along with iterations for the four algorithms, respectively. They objectively show that the performance of the new algorithm is better than those of the MLEM-TV, MRP, and MLEM algorithms.

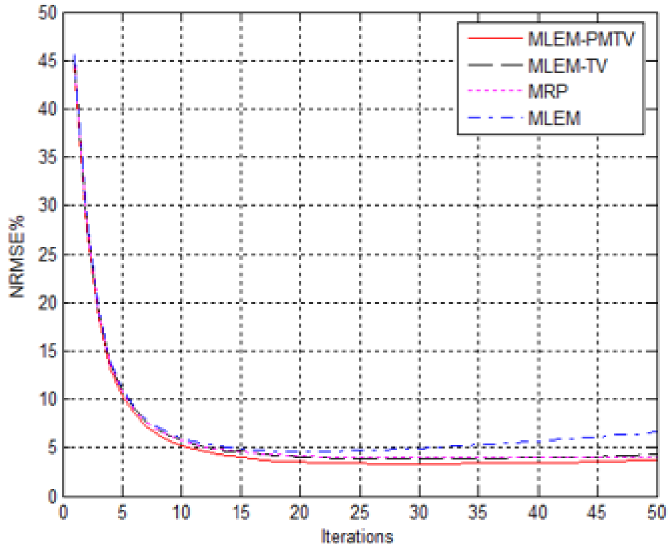

Figure 9. The plots of NRMSE along with iterations for thorax phantoms reconstructed by four algorithms.

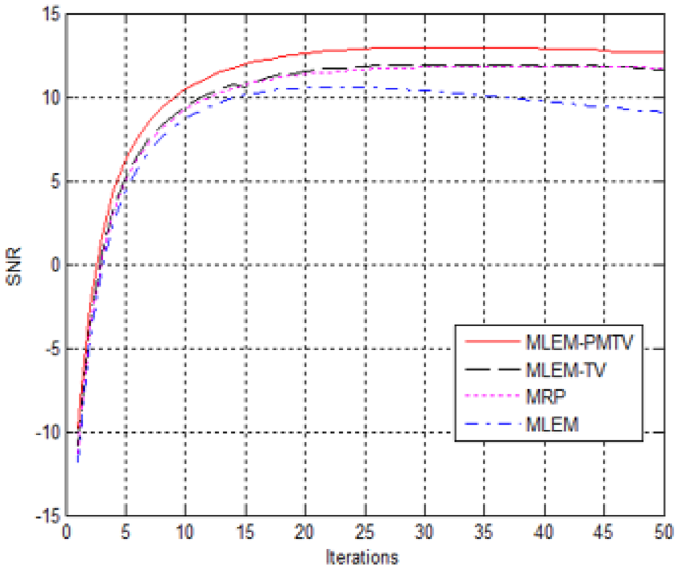

Figure 10. The plots of SNR along with iterations for thorax phantoms reconstructed by four algorithms.

7.CONCLUSION

In this paper, we proposed a new PML algorithm for PET image reconstruction by combining the Poisson-modified total variation model with the MLEM algorithm. The new method can improve the quality of reconstructed images. The simulation results show that the new algorithm absorbs the advantages of PMTV model, and can effectively suppress the Poisson noise in the reconstructed image. The new algorithm is a good compromise in two ways: noise suppression and edge protection. Moreover, the quality of the reconstructed images is greatly improved by this new method when compared to the presently used methods.

8. ACKNOWLEDGMENTS

The authors would like to thank anonymous reviewers for their helpful work of this paper. This work was supported by the Scientific research Foundation of Hunan Provincial Education Department under Grant No.15C0253.

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