Supply Chain Financial Ecosystem Analysis Based on Cusp Catastrophe Model

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Abstract

In light of the stability of supply chain financial system and its influencing factors, this paperuses ecological principles and methods to studysupply chain finance from the perspective of financial ecosystem, defines the concept of supply chain financial ecosystem, and applies the cusp catastrophe model to the stability evaluation of supply chain financial ecosystem. The author constructs the cusp catastrophe analysis model by defining the system variables through normative research, analyzes the four key variables affecting the working capital turnover through the empirical research, and examines the influence of the supply chain financial ecological environment on the corporate credit behavior. The results show that: When the bifurcation set of a statistical index satisfies the condition of less than zero, the balance of the variable is in the state of catastrophe; the improvement of the financial eco-environment of the supply chain helps to restrict the discreditable behavior of enterprises and improve the quality of the supply chain financial ecosystem. Finally, suggestions are made on how to strike a balance in and optimize the supply chain financial ecosystem.

Keywords: catastrophe theory, supply chain finance, influencing factors, ecosystem, working capital

1. INTRODUCTION

In the tide of globalization and market economy, supply chain management has become the mainstream industrial model, and the overall competition between different supply chains has become the main way of market competition. (Jin, 2005) The research focus on supply chain competitiveness has shifted from the logistics and information flow to the capital flow. (Barunik and Kukacka, 2015)In order to coordinate the flow of capital on the supply chain, supply chain finance came into being. It is of great theoretical and practical values to study the stability and influencing factors of supply chain finance (Zhao and Yang, 2013).

Supply chain finance has great potential in research, development and practical application because it creates an industrial ecology of mutually beneficial coexistence, sustainable development and benign interaction of bank, enterprise and commodity supply chain, a guarantee of the win-win of multiple parties on the supply chain. (Hardy, 1996) Supply chain finance integrates the capital resources throughout the supply chain, and enhances the competitiveness of the whole supply chain and even the entire industry. (Cobb, 2010) The theoretical and practical study of supply chain financial ecosystem play an important role in improving the domestic financial system and guiding the benign and sustainable development of the financial industry. (Dickinson, 1981) With unique advantages, the supply chain financial model attracts more and more banks. So far, it has already become a mainstream business model, vigorously promoted and developed by commercial banks. (Wimmerset al., 1998) Despite the rapid development, supply chain finance still faces many problems in practical application. (Sethi and King, 1998) For the purpose of optimizing and upgrading the entire supply chain financial system, all the parties involved in supply chain finance should make concerted efforts. (Barunik and Vosvrda, 2009)

In view of the above, this paper probes into supply chain finance from the perspective of financial ecosystem and selects the cusp catastrophe analysis of the supply chain financial ecosystem as the research direction. By constructing the cusp catastrophe model of supply chain financial ecosystem, the author identifies the factors affecting the catastrophe of supply chain financial ecosystem, and analyzes the influence of supply chain financial ecosystem on the credit behavior of enterprises. In this way, the author puts forward a new method for examining the changes to supply chain financial ecosystem, which has a positive effect on the future research on China's supply chain financial ecology.

2. THE PRINCIPLE OF CUSP CATASTROPHE THEORY

In the catastrophe theory, (Grasmanet al., 2009) a potential function is constructed first. The standard form is:

$$V(x) = \frac{1}{4}x^4 + \frac{1}{2}px^2 + qx \tag{1}$$

Where x is the system state variable; p and q are system control variables. When the first order derivative of $V(x)$ is zero, it is the system equilibrium equation (Figure 1a) called the equilibrium surface M. It is a smooth surface with folds, which represents the set of all critical points of the system.

$$V'(x) = x^3 + px + q = 0 \tag{2}$$

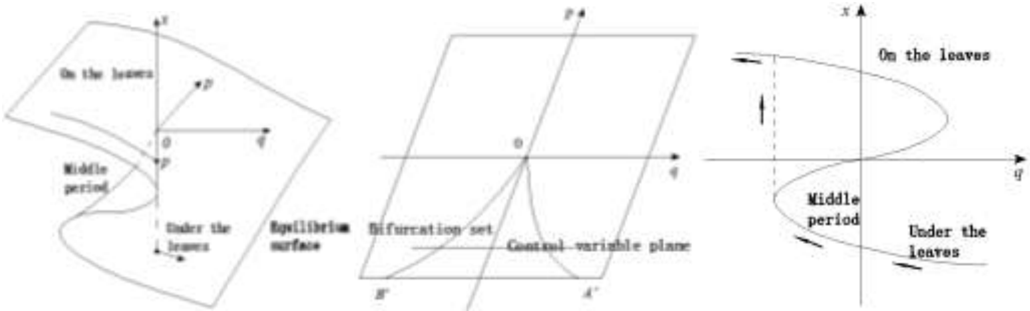
Find the second order derivative of the potential function to obtain the singularity equation (Smerzet al., 2008).

$$V''(x) = 3x^2 + p = 0 \tag{3}$$

Rearrange equations (2) and (3) to eliminate x to obtain the bifurcation set equation (8) of the system. The bifurcation set is the set of all the points that cause the state variable x to jump (Yokohiraet al., 1986).(Figure 1b)

$$4p^3 + 27q^2 = 0 \tag{4}$$

On the upper and lower leaves of Figure 1(a), $V''(x)>0$, the potential function takes the minimum value, and the equilibrium state is stable. On the middle leaf, $V''(x)<0$, the potential function takes the maximum value, and the equilibrium state is unstable. At the junction of the upper, lower and middle leaves of the surface, i.e. the smooth folds *OA* and *Ob* in Figure 1(b), $V''(x)=0$, the surface is at the critical state and satisfies the bifurcation set equation (4). With the effect of external load, the stable equilibrium point of the system gradually moves to the folds, and reaches the critical equilibrium state. However, the point will move to the unstable state of the middle leaf under the action of minor disturbance. The shift will cause the upper leaf to jump and result in the loss of stability of the system (Li et al., 2009). (Figure 1c)



(a) (b) (c)

Figure 1. Cusp catastrophe model

3. CATASTROPHE ANALYSIS OF SUPPLY CHAIN FINANCIAL ECOSYSTEM

3.1 Company background

EF is a private enterprise engaged in steel processing. At the end of 2015, it reaped a net profit of RMB 8.44 million yuan. The company achievedleapfrog development in 2016, making a net profit of RMB 16.89 million yuan (Guo et al., 2008). As shown in Table 1, most of the growth in the current assets of the company relies on the rapid increase in prepayments and accounts receivable. In contrast, the monetary capital decreases rapidly and the amount of stockremain largely the same. Thanks to the stable supply chain trading relationship, EF has achieved rapid growth in the short term. However, the long-term development of the company is constrained by the large amounts of accounts receivable and prepayments, (Tim et al., 2008) and the severe cash flow pressure. As the volume of business increases, there is a swift rise in various non-cash current assets (Chowet al., 2008).

Table 1 Current assets structure in the research period (Unit: RMB 10^4 yuan)

Year	2015/12	2016/12
Prepayments	166	1225
Accounts receivable	656	1770
Monetary fund	140	4
Stock	1555	1556

3.2 EF's position and influence in the supply chain

The EF company faces a severe shortage of circulating fund. Located in the middle of China's steel supply chain "iron ore-iron and steel production-steel processing-users", the company has deepened the cooperation with upstream and downstream enterprises, and formed a stable supply chain.

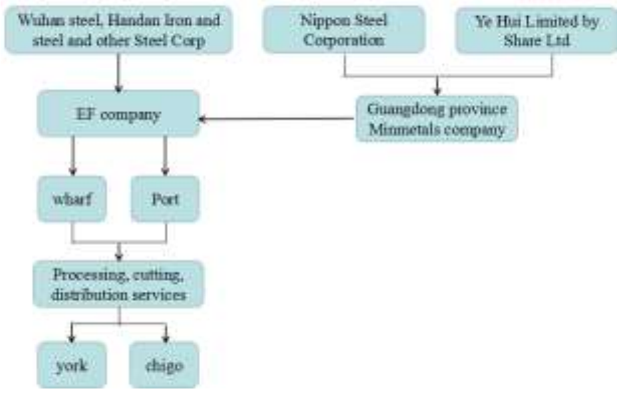


Figure 2. EF's position in the industry supply chain

EF company provides agent purchasing, processing and supporting services to the upstream and downstream enterprises. The procurement and sales procedures are shown in Figure 2. In the above transactions, the company is in a disadvantageous trading status: (1) During the procurement of sheets and plates, it not only has to pay the deposit, but also make the full payment for the goods after signing the supply contract; (2) During the sales of processed products, the company must allow the clients to make deferred payments, aiming at attracting clients and maintaining the sales channels, in which a considerable amount of accounts receivable is incurred.

Table 2. Assets structure of EF in the research period (Unit: RMB 104 yuan)

Year	2011/12	2012/12	2013/12	2014/12	2015/12
Total assets	5423	6802	6839	7020	7219
Including: Monetary Fund	317	313	113	2430	2041
Accounts receivable	1407	599	1605	1465	1683
Prepayments	1836	3329	1187	1381	1502
Other receivables	7	0.5	0.6	3.8	3
Stock	1835	2542	3908	1766	1963
Total current assets	5402	6784	6814	6996	7193
Long-term investment	0	0	0	0	0
Fixed assets	21	18	25	25	27

In 2011, AB Commercial Bank provided EF with a supply chain financial product in prepayment financing mode, which is targeted at the commodity procurement phase. The bank evaluated the asset allocation, working capital gap, and line of credit of EF, and decided to loan RMB 2.1 million yuan to the company for a year. Figure 3 illustrates the specific workflow of the business.

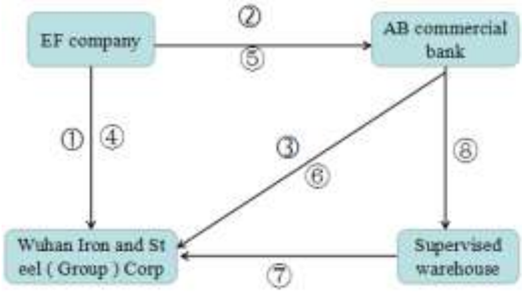


Figure 3. The workflow of prepayment financing mode

In 2012, EF and YORK® concluded a sales partnership contract. Soon, YORK® became one of EF's largest downstream clients. 2003 saw the stable growth of the trading volume and transaction amount between the two parties. Relying on its own capital, it is impossible for EF to maintain client relationship or achieve sustainable operation. Because of the above situation, EF filed an application to AB for the "accounts receivable financing" business. Figure 4 illustrates the specific workflow of the business.

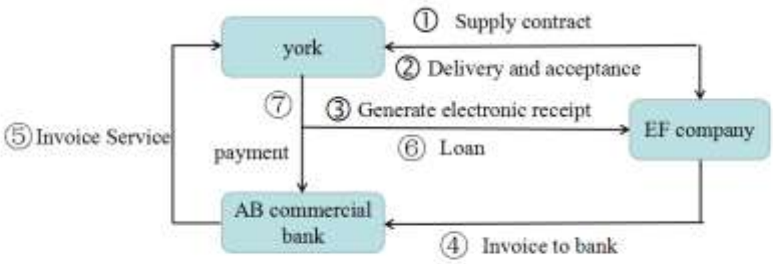


Figure 4. The workflow of accounts receivable financing mode

4. CUSP CATASTROPHE ANALYSIS BASED ON THE FOUR VARIABLES OF EF

4.1 Catastrophe analysis of the prepayments balance

Since the cusp catastrophe is a quadratic function, the quadratic function is used to fit the data. Take the time as the independent variable x (2009 is set as the benchmark and the first year, and the following years are deducted by analogy), take the prepayments balance as function Y_1, and uses quartic polynomial to fit the data in the following table by the least squares method. (Table 3)

Table 3 Fitting of the prepayments balance

x	1	2	3	4	5	6	7
Y_1	166	1225	1836	3329	1187	1381	1502

The resulting quartic polynomial is:

$$Y_1(x) = 36.97x^4 - 540.65x^3 + 2418.04x^2 - 3080.99x + 1340.57 \quad (5)$$

The results of the fitting are shown in the following Table (4):

Table 4. Fitting results of prepayments index

Year	2009	2010	2011	2012	2013	2014	2015
Prepayments account (actual)	166	1225	1836	3329	1187	1381	1502
Prepayments account(fitting)	173	1117	2256	2567	1911	1036	1579
Relative error: %	4.78	8.81	22.93	22.86	61.05	24.92	5.17

Table (4) displays the fitting results of prepayments balance with quartic polynomial. The relative error changes in a wide range from the maximum error of 61.05% to the minimum error of 4.78%.The fitting effect is satisfactory from 2009 to 2010, when the relative error is relatively small. However, the fitting effect is not ideal from 2011 to 2014. In 2008, the relative error is particularly high, which reaches 61.05%. Thus, we are not satisfactory with the fitting effect. Following is a catastrophe analysis of the prepayments balance data:

In the practical application, the potential equation of the cusp catastrophe model is obtained through elementary transformation of the quadratic polynomial or equation. Let the quadratic polynomial be:

$$y(m) = a_0 + a_1 m + a_2 m^2 + a_3 m^3 + a_4 m^4 \qquad (6)$$

To eliminate the cubic term in Formula (6), let $m = x - n$ and $n = a_3/4a_4$. Thus, Formula (6) is changed to:

$$F(x) = b_0 + b_1 x + b_2 x^2 + b_4 x^4 \qquad (7)$$

Make further variable substitution, and let $V(x) = F(x)/b$. There is:

$$V(x) = x^4 + ux^2 + vx + c$$

This is the standard form of the cusp catastrophe model. Make elementary transformation of Formula (5) to obtain:$n = a_3/4a_4 = -3.7$, $b_0 = 2591$, $b_1 = 135$, $b_2 = -545.9$ and $b_4 = 37$

$$F(x) = 2592 + 136x - 546x^2 + 37x^4$$

Make variable substitution with $V(x)=F(x)/37$. We obtain:

$$u = -15, v = 3.7, c = 70$$

Thus, the cusp catastrophe model is:

$$V(x) = x^4 - 15x^2 - 3.7x + 70$$

Calculate the bifurcation set for the cusp catastrophe model:

$$B = 8u^3 + 27v^2 = 8 \times (-15)^3 + 27 \times (-3.7)^2 < 0$$

Through the catastrophe analysis, it is found that the bifurcation set △<0. It can be seen that EF's prepayments balance data enters thestate of catastrophe in 2006.

4.2 Catastrophe analysis of the balance of accounts receivable

The analysis uses the same method as the analysis of prepayments balance. Take the time as the independent variable x (2009 is set as the benchmark and the first year, and the following years are deducted by analogy), take the balance of accounts receivable as function Y_2, and uses quartic polynomial to fit the data in the following table by the least squares method. (Table 5)

Table 5 Fitting of the accounts receivables index

x	1	2	3	4	5	6	7
$\boldsymbol{Y_2}$	656	1770	1407	599	1605	1465	1683

The resulting quartic polynomial is:

$$Y_2(x) = -34.17x^4 + 578.29x^3 - 3328.5x^2 + 7493.61x - 4032.14 \quad (8)$$

The results of the fitting are shown in the following table:

Table 6 Fitting results of the balance of accounts receivable

Year	2009	2010	2011	2012	2013	2014	2015
Prepayments account (actual)	656	1770	1407	599	1605	1465	1683
Prepayments account (fitting)	677	1720	1338	949	1153	1729	1638
Relative error: %	3	3	5	59	28	18	3

The above Table (6) displays the fitting results of the balance of accounts receivable with quartic polynomial. It is discovered that the fitting curve and the original curve share similar variation trends. The fitting effect is satisfactory from 2009 to 2010, when the relative error changes in a small range from 2.79% to 4.89%. However, the relative error is as high as 58.49% in 2012, making the fitting effect far from ideal.

According to the methods described in Formulas (6) &(7), make elementary transformation of the quartic polynomial equation (8) to obtain:

$$n = a_3 / 4a_4 = -4.2, b_0 = 949.9,\ b_1 = 26,\ b_2 = 342.2 \text{ and } b_4 = -34.2$$

$$F_2(x) = 949.9 + 26x + 342.2x^2 - 34.2x^4$$

Make variable substitution with*V(x)=F(x)/34.2*. We obtain:

$$u = -10.01, v = -0.76, c = -27.8$$

Thus, the cusp catastrophe model is:

$$V(x) = x^4 - 10.01x^2 - 0.76x + 27.8$$

Calculate the bifurcation set for the cusp catastrophe model:

$$B = 8u^3 + 27v^2 = 8 \times (-10.01)^3 + 27 \times (-0.76)^2 < 0$$

Analyzing the cusp catastrophe model, it is found that the balance of accounts receivable of EF enters the state of catastrophe in 2007. From 2012 to 2015, the relative error gradually decreases, indicating that the balance of accounts receivable of EF begins to shift from the state of catastrophe to the state of equilibrium in 2012.

4.3 Catastrophe analysis of the balance of monetary fund

This section is about the catastrophe analysis of the balance of monetary fund. Take the time as the independent variable *x* (2009 is set as the benchmark and the first year, and the following years are deducted by analogy), take the balance of monetary fund as function Y_3, and uses quartic polynomial to fit the data in the following table by the least squares method. (Table 7)

Table 7 Fitting of monetary fundindex

x	1	2	3	4	5	6	7
$\boldsymbol{Y_3}$	140	4	317	313	113	2430	2041

$$Y_3(x) = -31x^4 + 487x^3 - 2474x^2 - 4772x - 2706 \quad (9)$$

Table 8 Fitting results of monetary fundindex

Year	2009	2010	2011	2012	2013	2014	2015
Prepayments account (actual)	1555	1556	1835	2542	3908	1766	1963
Prepayments account (fitting)	1653	1206	2120	2951	3010	2361	1824
Relative error: %	6	23	16	16	23	34	7

Table (8) displays the fitting results of prepayments balance with quartic polynomial. It can be seen that the fitting effect is very unsatisfactory due to the extremely high relative error. Analysis of the cusp catastrophe model demonstrates that the monetary fund data of EF is in the state of catastrophe in any year except for 2011 and 2012, when it is in the state of equilibrium. The cusp catastrophe analysis goes as follows:

According to the methods described in Formulas (6) & (7), make elementary transformation of the monetary fund equation (9) to obtain:

$$n = a_3 / 4a_4 = -3.93,\ b_0 = 33.52,\ b_1 = 369.7,\ b_2 = 398.44,\ b_4 = -31.01$$

$$F_2(x) = 33.52 + 369.7x + 398.44x^2 - 31.01x^4$$

Make variable substitution with *V(x)=F(x)/31.01*. We obtain:

$$u = -12.58, v = -11.92, c = -1.08$$

Thus, the cusp catastrophe model is:

$$V(x) = x^4 - 12.58x^2 - 11.92x - 1.08$$

Calculate the bifurcation set for the cusp catastrophe model:

$$B = 8u^3 + 27v^2 = 8 \times (-12.58)^3 + 27 \times (-11.92)^2 < 0$$

4.4 Catastrophe analysis of the stock balance

Take the time as the independent variable *x* (2009 is set as the benchmark and the first year, and the following years are deducted by analogy), take the stock balance as function Y_4, and uses quartic polynomial to fit the data in the following table by the least squares method. (Table 9)

Table 9 Fitting of stock balanceindex

x	1	2	3	4	5	6	7
$\boldsymbol{Y_4}$	1555	1556	1835	2542	3908	1766	1963

The quartic polynomial is obtained as follows:

$$Y_4(x) = 31x^4 - 554^2 + 3224x^3 - 6706x + 5659 \quad (10)$$

See the table below for the fitting results:

Table 10 Fitting results of stockbalanceindex

Year	2009	2010	2011	2012	2013	2014	2015
Prepayments account (actual)	1555	1556	1835	2542	3908	1766	1963
Prepayments account (fitting)	1653	1206	2120	2951	3010	2361	1824
Relative error: %	6	23	16	16	23	34	7

In accordance with the fitting results in Table (10), the relative error changes in a relatively wide range from the maximum value of 34% to the minimum value of 6%. Thus, the fitting effect is not so satisfactory. However, the relative error of the stock balance varies in a much narrower range than that of prepayments balance, the balance of accounts receivable, and the balance of monetary fund.

Following is the catastrophe analysis of the statistics on the stock balance. According to the methods described in Formulas (6) & (7), make elementary transformation of the stock balance equation (10) to obtain:

$$n = a_3 / 4a_4 = -4.41,\ b_0 = 3086,\ b_1 = 134.6,\ b_2 = -449.5 \text{ and } b_4 = 31.42$$

$$F_2(x) = 3085.93 + 134.64x - 449.54x^2 + 31.42x^4$$

Make variable substitution with*V(x)=F(x)/31.42*. We obtain:

$$u = -14.31, v = 4.29, c = 98.22$$

Thus, the cusp catastrophe model is:

$$V(x) = x^4 - 14.31x^2 + 4.29x + 98.22$$

Calculate the bifurcation set for the cusp catastrophe model:

$$B = 8u^3 + 27v^2 = 8 \times (-14.31)^3 + 27 \times (4.29)^2 < 0$$

The fitting effect is rather poor for the relative error of EF’s stock balance in 2009 reaches 33.72%. In light of the above catastrophe model analysis, it is concluded that the EF’s stock balance is in the state of catastrophe in 2009. Nevertheless, the stock balance varies in a much narrower range than the balances of other three items throughout the period from 2004 to 2010.

4.5 Principal component analysis of the four variables

Since the working capital of small and medium-sized enterprises (SMEs) is mainly used for prepayments, accounts receivable, stock and monetary funds, the balances of the four indices are critical to the operating capacity of SMEs. Hence, the principal component analysis is employed to process the data of the four variables (Table 11) between 2004 and 2010, and to synthesize the data into two consolidated balances.

Table 11 Assets structure of EF company from 2009 to 2015 (Unit: RMB 10^4 yuan)

Year	2009/12	2010/12	2011/12	2012/12	2013/12	2014/12	2015/12
Prepayments	166	1225	1836	3329	1187	1381	1502
Accounts receivable	656	1770	1407	599	1605	1465	1683
Monetary Fund	140	4	317	313	113	2430	2041
Stock	1555	1556	1835	2542	3908	1766	1963

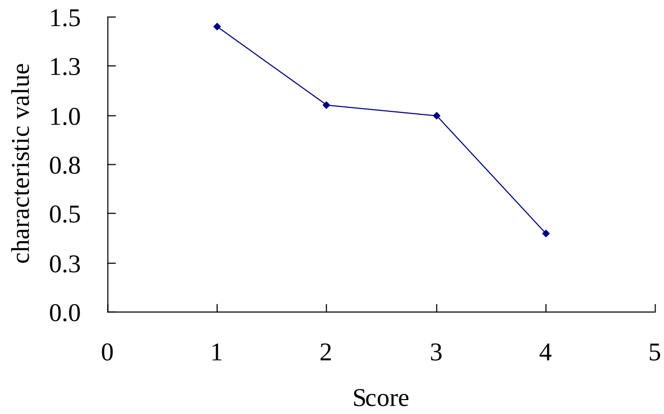


Figure 5 Scree plot

In light of scree plot (5), 3 principal components should be selected in the principle that the eigenvalue should be greater than 1. Considering the fact that the sum of variance of the first two principal components accounts for 99.8% of the total variance, i.e. the two components cover 99.8% of the total information of the original data, it is enough to summarize most of the information of the original data is summarized with the first two principal components. In replacement of the four balances, the first two principle components make an accurate description of the working capital turnover of the enterprise. We denote the two principal components by *u* and *v*. Based on the above tables, the linear combination of *u* and *v* are as follows:

$$u = -0.558x_1 + 0.673x_2 + 0.681x_3 - 0.523x_4$$

$$v = 0.24x_1 + 0.653x_2 + 0.168x_3 + 0.758x_4$$

Substitute the expressions of *u* and *v* into the potential function equation of the cusp catastrophe model, there is:

$$\begin{aligned} &4(-0.558x_1 + 0.637x_2 + 0.681x_3 + 0.523x_4)^3 \\ &+ 27(0.24x_1 + 0.653x_2 + 0.168x_3 + 0.7585x_4)^3 \end{aligned} \tag{11}$$

This is the condition for the model to enter the state of catastrophe. The working capital in a state of mutation if the values of variables like the prepayments, accounts receivable, stock and monetary fund satisfy Formula (11); otherwise, the working capital is generally in the state of equilibrium.

The results of the above catastrophe analysis are consistent with the actual situation in the development of EF. This means that the cusp catastrophe model is applicable to the catastrophe analysis of the four variables on the working capital of EF, and the conclusion of the analysis is viable. Facing the gap in working capital, EF should make timely use of the financing tool of supply line finance to ease the pressure of circulating fund and avoid the risk of capital chain interruption. The supply chain finance not only provides a guarantee for the stable development of the company, but also forcefully enhances the relationship between the upstream and downstream enterprises. The supply chain finance system is trending towards the state of steady circulation.

4. CONCLUSION

(1) This paper constructs a cusp catastrophe model for supply chain financial ecosystem. It is discovered that the function of the supply chain financial system has not been achieved when the two control variables E and P are in a poor state. The improvement of the supply chain financial environment E can effectively curb the discreditable behavior of the SMEs, and prevent the instability of the enterprise behavior. In this case, the quality of the entire supply chain financial ecosystem will be improved, and the supply chain financial system will be stabilized and optimized.

(2) The flexible use of supply chain finance has many advantages. It helps SMEs to resolve the shortage of funds, and ensures the rational distribution of capital flows in the supply chain and the smooth cooperation among all parties. In addition, it can effectively improve the financial situation of the entire supply chain and save the financial cost and the cost of final products of the supply chain. All in all, supply chain finance is a guarantee of the win-win of multiple parties, a boost to the stabilization and upgrading of the supply chain financial ecosystem, and a driver of the upgrading and transformation of the industry.

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